Investigation 2

Whole-Number Patterns and Relationships

Because you have been using whole numbers since you were young, you may think there is not much more to learn about them. However, there are many interesting relationships involving whole numbers that you may never have considered. To notice these relationships, it is sometimes helpful to break whole numbers into factors or to multiply them by other numbers.

2.1 Finding Patterns

In the Factor Game and the Product Game, you found that factors occur in pairs. Once you know one factor of a number, you can find another factor. For example, 3 is a factor of 12, and because $3 \times 4 = 12$, 4 is also a factor of 12. We call the pair 3, 4 a **factor pair.**

Every year, Meridian Shopping Mall has an exhibit of arts and crafts. People who want to display their work rent a space for $20 per square yard. Exhibitors are given carpet squares to lay out their spaces. Each carpet square measures 1 square yard. All exhibit spaces must have a rectangular shape.

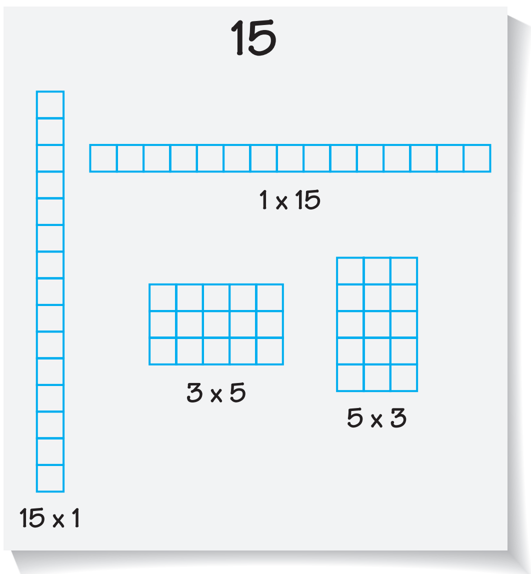
Getting Ready for Problem 2.1

Terrapin Crafts wants to rent a space of 12 square yards. Use 12 square tiles to represent the carpet squares.

- What are all the possible ways the Terrapin Crafts owner can arrange the squares to make a rectangle?
- How are the rectangles you found and the factors of 12 related?

You just found all the possible rectangles that can be made from 12 tiles. These rectangles are *models* for the number 12. The models are useful because they allow you to "see" the factors of 12. You can make rectangle models such as these for any whole number.

In Problem 2.1, you and your classmates will use grid paper to create all the possible rectangle models for all the whole numbers from 1 to 30. When the rectangles are displayed, you can look for interesting patterns.



Your teacher will assign your group a few of the numbers from 1 to 30. Work with your group to decide how to distribute the numbers you have been assigned.

Problem 2.1 Rectangles and Factor Pairs

A. From grid paper, cut out all the possible rectangle models you can make for each of your numbers. You may want to use tiles to help you find the rectangles.

Write each number at the top of a sheet of paper, and tape all the rectangles for that number to the sheet. List the factors of the number from least to greatest at the bottom of the paper.

Display the sheets of rectangles in order from 1 to 30 around the room. When all the numbers are displayed, look for patterns.

B. **1.** Which numbers have the most rectangles? What kind of numbers are these?

2. Which numbers have the fewest rectangles? What kind of numbers are these?

3. Which numbers are **square numbers** (numbers whose tiles can be arranged to form a square)?

4. How can you use the rectangle models for a number to list the factors of the number? Use an example to show your thinking.

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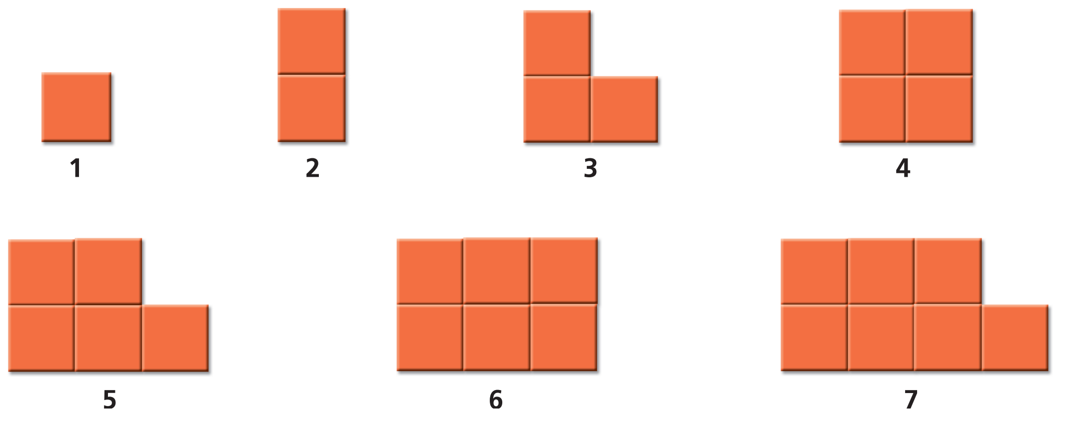
2.2 Reasoning With Even and Odd Numbers

An **even number** is a number that has 2 as a factor. An **odd number** is a number that does not have 2 as a factor.

Getting Ready for Problem 2.2

Tilo makes models for whole numbers by arranging square tiles in a special pattern. Here are Lilo's tile models for the numbers from 1 to 7.



- How are the models of even numbers different from the models of odd numbers?
- Describe the models for 50 and 99.

When you tell what you think will happen in a mathematical situation, you are making a conjecture. A **conjecture** is your best guess about a pattern or a relationship that you observe. You can use models, drawings, or other kinds of evidence to support your conjectures.

Make a conjecture about what happens when you add two even numbers. Do you get an even number or an odd number? Why?

Problem 2.2 asks you to think of other conjectures to make about even and odd numbers.

Problem 2.2 Reasoning With Even and Odd Numbers

A. Make conjectures about whether the results below will be even or odd. Then use tile models or some other method to support your conjectures.

1. the sum of two even numbers
2. the sum of two odd numbers
3. the sum of an even number and an odd number
4. the product of two even numbers
5. the product of two odd numbers
6. the product of an even number and an odd number

B. Is 0 an even number or an odd number? How do you know?

C. Without building a tile model, how can you determine whether a sum of numbers, such as $127 + 38$, is even or odd?

D. A problem occurs when we compute $6 + 3 \times 9$. You can get 81 or 33 as the answer! How can you get 81? How can you get 33? The *order of operations* rule says that you do all multiplications and divisions before you add or subtract. This makes 33 the correct answer.

Compute each number and tell whether it is even or odd.

1. $3 + 5 \times 7$ **2.** $25 - 3 \times 2$ **3.** $11 \times 5 + 3 \times 9$

4. $43 - 25 \div 5 + 2$ **5.** $43 - 7 + 5 \times 2$ **6.** $6 + 18 - 24 \div 6$

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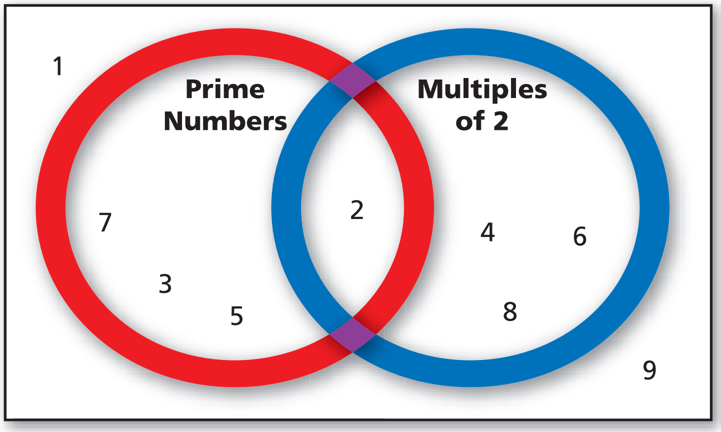
2.3 Classifying Numbers

A **Venn diagram** uses circles to group things that belong together. You can use Venn diagrams to explore relationships among whole numbers. For example, suppose that you want to group the whole numbers from 1 to 9 according to whether they are prime or multiples of 2. First, list the numbers that fall into each category:

Prime Numbers: 2, 3, 5, 7 Multiples of 2: 2, 4, 6, 8

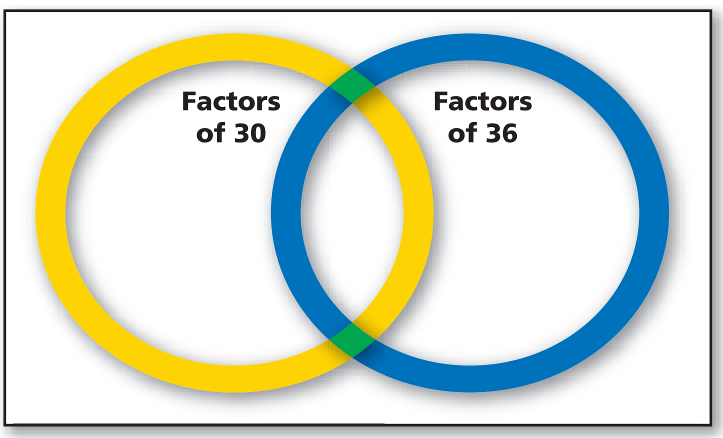
Next, draw and label two overlapping circles, one that represents the prime numbers and one that represents the multiples of 2. Put each number from 1 to 9 in the appropriate region. The numbers that don't fall into either category belong outside of the circles. The numbers that are in both categories belong in the overlap of the circles.



Problem 2.3 Classifying Numbers

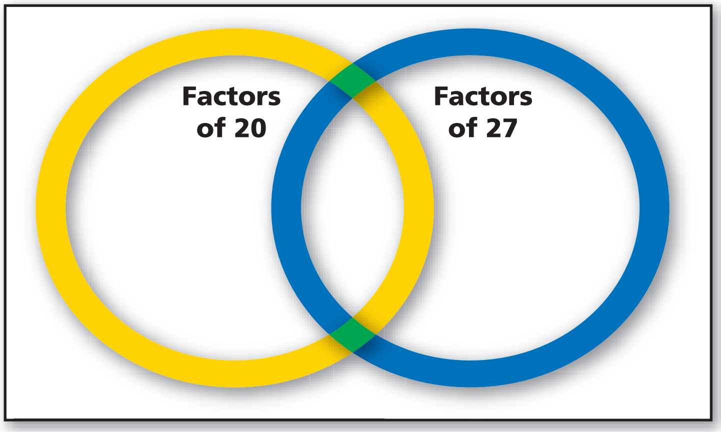
The Venn diagrams in Questions A–D are related to the ideas you studied in Investigation 1.

A. List the factors of 30 and 36. Fill in a copy of this Venn diagram with all whole numbers less than or equal to 40. Then answer the questions below.



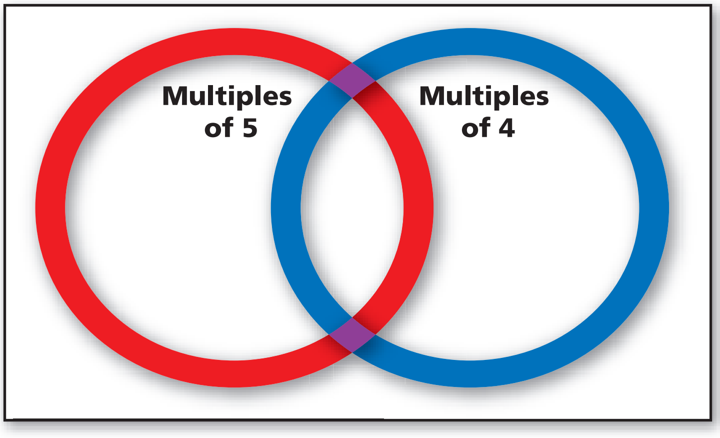
1. What do the numbers in the intersection (the "overlap") of the circular regions have in common?
2. List five numbers that fall in the region outside the circles and explain why they belong outside the circles.
3. Explain how you can use your completed diagram to find the greatest factor that 30 and 36 have in common. What is this *greatest common factor*?
4. What is the least number that falls in the intersection?

B. List the factors of 20 and the factors of 27. Fill in a copy of this Venn diagram with whole numbers less than or equal to 30. Then answer the questions below.



1. What do the numbers in the intersection of the circular regions have in common?
2. Explain how you can use your completed diagram to find the greatest factor that 20 and 27 have in common. What is this greatest common factor?
3. Compare this Venn diagram to the one you completed in Question A. How are they alike, and how are they different?

C. List the multiples of 5 and the multiples of 4 that are less than or equal to 40. Fill in a copy of this Venn diagram with whole numbers less than or equal to 40. Then answer the questions that follow.

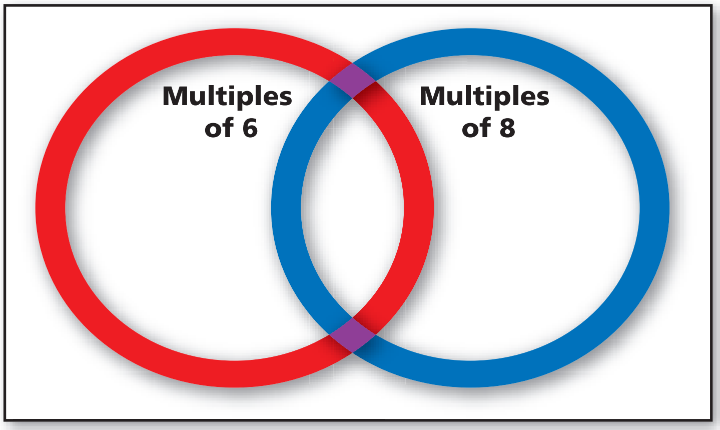


1. What do the numbers in the intersection of the circular regions have in common?

2. Explain how you can use your completed diagram to find the least multiple that 5 and 4 have in common. What is this *least common multiple*?

3. List five more numbers that would be in the intersection if numbers greater than 40 were allowed. What would be the greatest possible number in the intersection if you could use any number?

D. List the multiples of 6 and the multiples of 8 that are less than or equal to 48. Fill in a copy of this Venn diagram with whole numbers less than or equal to 48. Then answer the questions below.



1. What do the numbers in the intersection have in common?

2. Explain how you can use your completed diagram to find the least multiple that 6 and 8 have in common. What is this least common multiple?

3. Compare this Venn diagram to the one you completed in Question C. How are they alike? How are they different?

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