Investigation 1

Factors and Products

Jamie is 12 years old. Her cousin, Emilio, is 2 years old. Her brother, Cam, is 3. Her neighbor, Esther, is 8. The following number sentences say that Jamie is

6 times as old as Emilio, 4 times as old as Cam, and $1\frac{1}{2}$ times as old as Esther.

$12 = 6 \times 2$ $12 = 4 \times 3$ $12 = 1\frac{1}{2} \times 8$

Notice that each of the whole numbers 2, 3, 4, and 6 can be multiplied by another whole number to get 12. For this reason, 2, 3, 4, and 6 are called *whole-number factors*, or *whole-number divisors*, of 12.

Although 8 *is* a whole number, it *is not* a whole-number factor of 12 because you cannot multiply 8 by another whole number to get 12.

To save time, we will simply use the words **factor** and **divisor** to refer to whole-number factors and whole-number divisors of a number.

1.1 Playing the Factor Game

Playing the Factor Game is a fun way to practice finding factors of whole numbers. If you pay close attention, you may learn some interesting things about numbers that you didn't know before! To play the game, you need a Factor Game Board and colored pens, pencils, or markers.

active math online
For: Factor Game Activity
Visit: PHSchool.com
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The Factor Game

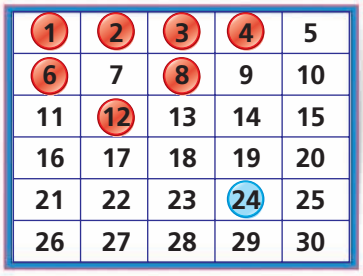
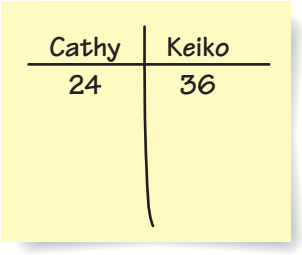
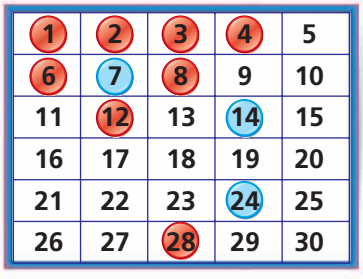
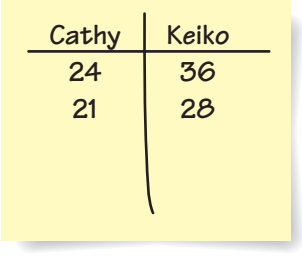
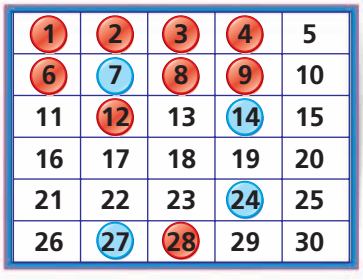
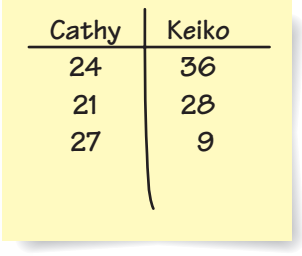
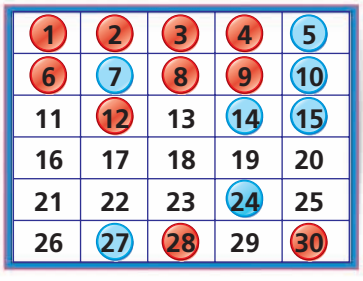
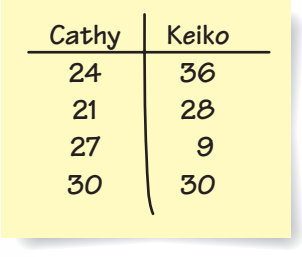
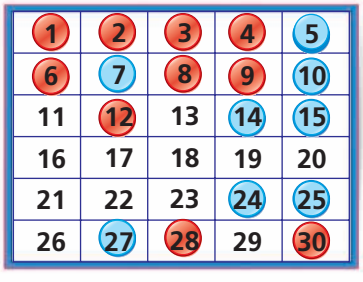
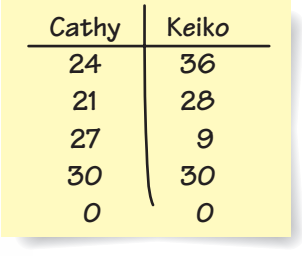
1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30

Factor Game Rules

1. Player A chooses a number on the game board and circles it.
2. Using a different color, Player B circles all the proper factors of Player A's number. The **proper factors** of a number are all the factors of that number, except the number itself. For example, the proper factors of 12 are 1, 2, 3, 4, and 6. Although 12 is a factor of itself, it is not a proper factor.
3. Player B circles a new number, and Player A circles all the factors of the number that are not already circled.
4. The players take turns choosing numbers and circling factors.
5. If a player circles a number that has no factors left that have not been circled, then that player does not get the points for the number circled and loses the next turn.
6. The game ends when there are no numbers left with uncircled factors.
7. Each player adds the numbers that are circled with his or her color. The player with the greater total is the winner.

The First Five Moves of a Sample Game

This table shows the first five moves of a game between Cathy and Keiko. The first column describes the moves the players made. The other columns show the game board and the score after each move.

Action	Game Board	Score
Cathy circles 24. Keiko circles 1, 2, 3, 4, 6, 8, and 12 (the proper factors of 24).		
Keiko circles 28. Cathy circles 7 and 14 (the factors of 28 that are not already circled).		
Cathy circles 27. Keiko circles 9 (the only factor of 27 that is not already circled).		
Keiko circles 30. Cathy circles 5,10 and 15 (the factors of 30 that are not already circled).		
Cathy circles 25. All the factors of 25 are circled. Cathy does not receive any points for this turn and loses her next turn.		

Problem 1.1 Finding Proper Factors

A. Play the Factor Game several times with a partner. Take turns making the first move. Look for moves that will give you more points than your opponent. As you play, write down any strategies or patterns you find.

B. How can you test to determine whether a number is a factor of another number?

C. If you know a factor of a number, can you find another factor? Explain your thinking.

D. Give an example of a number that has many factors and an example of a number that has few factors.

E. Make a list of the factors of 18. Make a list of the divisors of 18. Are the factors of a number also divisors of the number? Explain your thinking.

F. How do you know when you have found all the factors of a number?

ACE Homework starts on page 14.

1.2 Playing to Win the Factor Game

Did you notice that some numbers are better than others to choose for the first move in the Factor Game? For example, if you choose 22, you get 22 points and your opponent gets only $1 + 2 + 11 = 14$ points. However, if you choose 18, you get 18 points, and your opponent gets $1 + 2 + 3 + 6 + 9 = 21$ points!

Now you will make a table to analyze the Factor Game and look for patterns. Your table might start like this:

First Move	Proper Factors	My Score	Opponent's Score
1	None	Lose a Turn	0
2	1	2	1
3	1	3	1
4	1, 2	4	3

Problem 1.2 Prime and Composite Numbers

A. **1.** Make a table of all the possible first moves (numbers from 1 to 30) in the Factor Game.

2. For each move, list the proper factors of the number, and record the scores you and your opponent would receive.

3. Describe an interesting pattern you see in your table.

B. What is the best first move? Why?

C. Which first move would make you lose your next turn? Why?

D. Other than your answer to Question C, what is the worst first move? Why?

E. List all the first moves that allow your opponent to score only one point. These numbers are called *prime numbers*.

F. Are all prime numbers good first moves? Explain. (Remember, a number is a *good first move* if the player choosing the number scores more points than his or her opponent.)

G. List all the first moves that allow your opponent to score more than one point. These numbers also have a special name. They are called *composite numbers*.

H. Are composite numbers good first moves? Explain.

ACE **Homework starts on page 14.**

Did You Know?

Large prime numbers are used to encode top-secret information. In 1999, Nayan Hajratwala found a prime number with more than 2 million digits. In type this size, that number would be more than 2 miles long! The Electronic Frontier Foundation awarded Mr. Hajratwala $50,000 for discovering the first prime number with more than 1,000,000 digits. The EFF now offers a prize of $250,000 to the first person to find a prime number with over 1,000,000,000 digits!

Mathematicians have always been puzzled about fast ways of determining whether really big numbers are prime. In August, 2002, Dr. Manindra Agrawal and two college students, Neeraj Kayal and Nitin Saxena, made a breakthrough. They surprised and delighted mathematicians with an elegant way of determining whether really huge numbers are prime. You can find more information about this in the August 8, 2002, issue of *The New York Times*.

Go Online PHSchool.com **For:** Information about prime numbers **Web Code:** ame-9031

1.3 The Product Game

You learned about factors of a number in Problems 1.1 and 1.2. In the next game you will learn about multiples of numbers. A **multiple** of a number is the product of that number and another whole number. For example, 24 is a multiple of 6 because $4 \times 6 = 24$. Multiples and factors have an interesting back-and-forth relationship.

If a number is a multiple of 5, then 5 is a factor of that number. These five sentences describe how the numbers 3, 5, and 15 are related.

$5 \times 3 = 15$

5 is a factor of 15.

3 is a factor of 15.

15 is a multiple of 5.

15 is a multiple of 3.

You can probably think of other ways to show the relationship. For example, you could add these to the list:

15 is divisible by 5.

15 is divisible by 3.

In the Factor Game, you start with a number and find its factors. In the Product Game, you start with factors and find their product. The Product Game board consists of a list of factors and a grid of products. The object is to mark four products in a row—up and down, across, or diagonally—before your opponent does.

active math online
For: Product Game Activity
Visit: PHSchool.com
Web Code: amd-1103

The Product Game

1	2	3	4	5	6
7	8	9	10	12	14
15	16	18	20	21	24
25	27	28	30	32	35
36	40	42	45	48	49
54	56	63	64	72	81

Factors:

1 2 3 4 5 6 7 8 9

To play the game, you need a Product Game Board, two paper clips, and colored markers or chips—one color for each player.

Product Game Rules

1. Player A puts a paper clip on a number in the factor list. Player A does not mark a square on the product grid because only one factor has been marked. It takes two factors to make a product.
2. Player B puts the other paper clip on any number in the factor list (including the same number marked by Player A). Player B then shades or covers the product of the two factors on the product grid. An example is shown on the next page.
3. Player A moves *either* paper clip to another number, leaving one in its original place, and then shades or covers the new product.
4. Each player, in turn, moves a paper clip and marks a product. If a product is already marked, the player does not get a mark for that turn. The winner is the first player to mark four squares in a row—up and down, across, or diagonally.

Problem 1.3 Finding Multiples

A. Play the Product Game several times with a partner. Look for interesting patterns and strategies that might help you win. Make notes on your observations.

B. Examine the Product Game Board. Is it possible to get every number on the product grid by multiplying two of the numbers in the factor list? Justify your answer.

C. Can you find two numbers in the list of factors for the game whose product is *not* on the product grid?

D. Suppose that a game is in progress and you want to cover the number 12 on the grid. Describe one way this can happen. Can you get 12 in more than one way?

E. **1.** Suppose that a game is in progress and one of the paper clips is on 5. What products can you make by moving the other paper clip?

2. List five multiples of 5 that are not on the game board.

ACE Homework starts on page 14.

The Product Game

1	2	3	4	5	6
7	8	9	10	12	14
15	16	18	20	21	24
25	27	28	30	32	35
36	40	42	45	48	49
54	56	63	64	72	81

Factors:

1 2 3 4 5 6 7 8 9