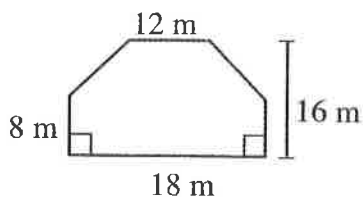
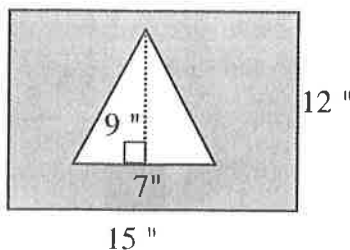


G

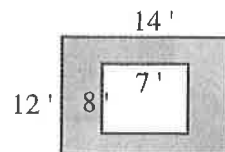
10.



11. Find the area of the shaded region.



12. Find the area of the shaded region.



**Answers**

- |                |                 |                   |                 |
|----------------|-----------------|-------------------|-----------------|
| 1. 158 sq. ft. | 2. 225 sq. m.   | 3. 303 sq. in.    | 4. 42 sq. yd.   |
| 5. 95 sq. m.   | 6. 172.5 sq. m. | 7. 252 sq. cm.    | 8. 310 sq. ft.  |
| 9. 23 sq. cm.  | 10. 264 sq. m.  | 11. 148.5 sq. in. | 12. 112 sq. ft. |

**PRISMS – VOLUME AND SURFACE AREA**

**SURFACE AREA OF A PRISM**

The surface area of a prism is the sum of the areas of all of the faces, including the bases. Surface area is expressed in square units.

For additional information, see the Math Notes boxes in Lessons 9.2.1 and 9.2.2 of the *Core Connections, Course 1* text and Lesson 9.2.4 of the *Core Connections, Course 2* text.

**Example**

Find the surface area of the triangular prism at right.

Step 1: Area of the 2 bases:  $2 \left[ \frac{1}{2} (6 \text{ cm})(8 \text{ cm}) \right] = 48 \text{ cm}^2$

Step 2: Area of the 3 lateral faces

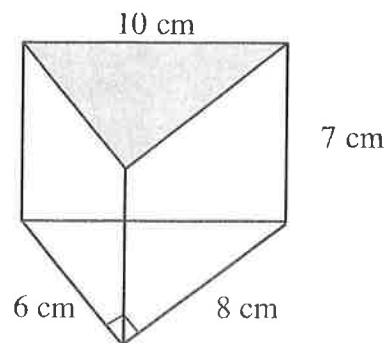
Area of face 1:  $(6 \text{ cm})(7 \text{ cm}) = 42 \text{ cm}^2$

Area of face 2:  $(8 \text{ cm})(7 \text{ cm}) = 56 \text{ cm}^2$

Area of face 3:  $(10 \text{ cm})(7 \text{ cm}) = 70 \text{ cm}^2$

Step 3: Surface Area of Prism = sum of bases and lateral faces:

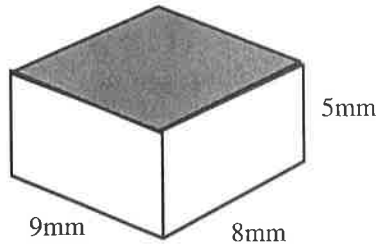
$SA = 48 \text{ cm}^2 + 42 \text{ cm}^2 + 56 \text{ cm}^2 + 70 \text{ cm}^2 = 216 \text{ cm}^2$



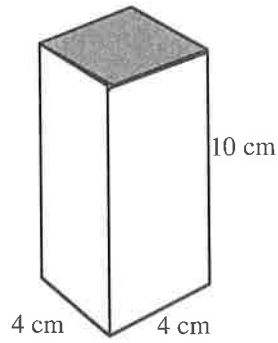
## Problems

Find the surface area of each prism.

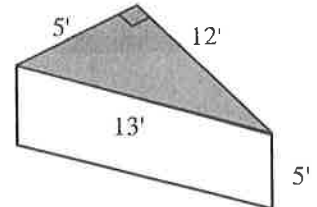
1.



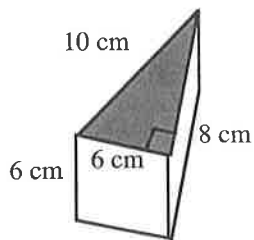
2.



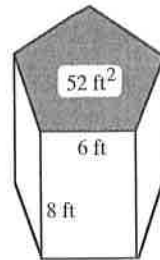
3.



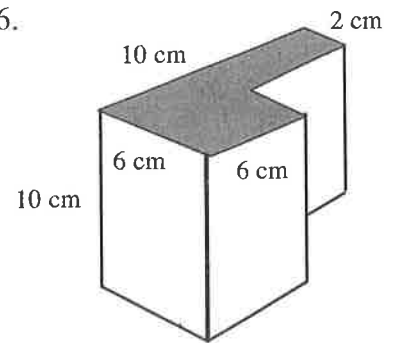
4.



5. The pentagon is equilateral.



6.



## Answers

1.  $314 \text{ mm}^2$

2.  $192 \text{ cm}^2$

3.  $210 \text{ ft}^2$

4.  $192 \text{ cm}^2$

5.  $344 \text{ ft}^2$

6.  $408 \text{ cm}^2$

## VOLUME OF A PRISM

Volume is a three-dimensional concept. It measures the amount of interior space of a three-dimensional figure based on a cubic unit, that is, the number of 1 by 1 by 1 cubes that will fit inside a figure.

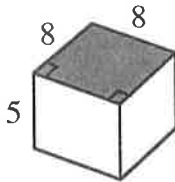
The volume of a prism is the area of either base ( $B$ ) multiplied by the height ( $h$ ) of the prism.

$$V = (\text{Area of base}) \cdot (\text{height}) \text{ or } V = Bh$$

For additional information, see the Math Notes boxes in Lesson 9.2.1 of the *Core Connections, Course 1* text and Lesson 9.2.4 of the *Core Connections, Course 2* text.

### Example 1

Find the volume of the square prism below.

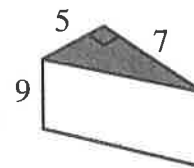


The base is a square with area ( $B$ )  
 $8 \cdot 8 = 64 \text{ units}^2$ .

$$\begin{aligned} \text{Volume} &= B(h) \\ &= 64(5) \\ &= 320 \text{ units}^3 \end{aligned}$$

### Example 2

Find the volume of the triangular prism below.

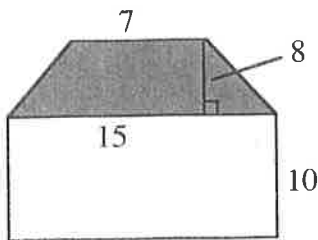


The base is a right triangle with area  
 $\frac{1}{2}(5)(7) = 17.5 \text{ units}^2$ .

$$\begin{aligned} \text{Volume} &= B(h) \\ &= 17.5(9) \\ &= 157.5 \text{ units}^3 \end{aligned}$$

### Example 3

Find the volume of the trapezoidal prism below.



The base is a trapezoid with area  
 $\frac{1}{2}(7 + 15) \cdot 8 = 88 \text{ units}^2$ .

$$\begin{aligned} \text{Volume} &= B(h) \\ &= 88(10) \\ &= 880 \text{ units}^3 \end{aligned}$$

### Example 4

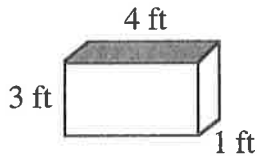
Find the height of the prism with a volume of  $132.5 \text{ cm}^3$  and base area of  $25 \text{ cm}^2$ .

$$\begin{aligned} \text{Volume} &= B(h) \\ 132.5 &= 25(h) \\ h &= \frac{132.5}{25} \\ h &= 5.3 \text{ cm} \end{aligned}$$

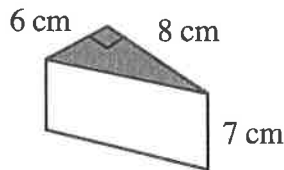
## Problems

Calculate the volume of each prism. The base of each figure is shaded.

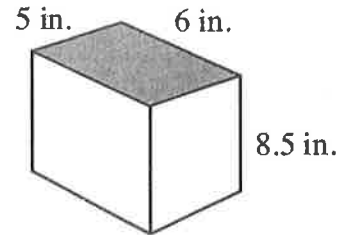
1. Rectangular Prism



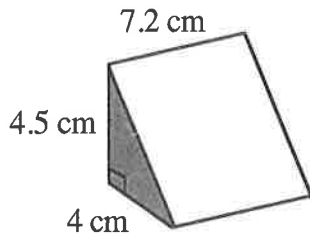
2. Right Triangular Prism



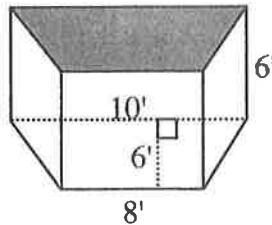
3. Rectangular Prism



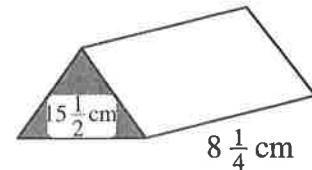
4. Right Triangular Prism



5. Trapezoidal Prism



6. Triangular Prism with  $B = 15\frac{1}{2}\text{ cm}^2$



7. Find the volume of a prism with base area  $32\text{ cm}^2$  and height  $1.5\text{ cm}$ .
8. Find the height of a prism with base area  $32\text{ cm}^2$  and volume  $176\text{ cm}^3$ .
9. Find the base area of a prism with volume  $47.01\text{ cm}^3$  and height  $3.2\text{ cm}$ .

## Answers

1.  $12\text{ ft}^3$       2.  $168\text{ cm}^3$       3.  $240\text{ in.}^3$       4.  $64.8\text{ cm}^3$       5.  $324\text{ ft}^3$
6.  $127\frac{7}{8}\text{ cm}^3$       7.  $48\text{ cm}^3$       8.  $5.5\text{ cm}$       9.  $14.7\text{ cm}^2$

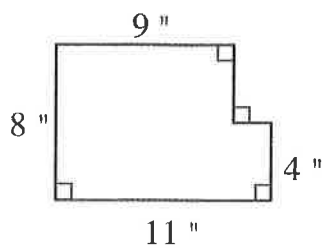
H

## CALCULATING COMPLEX AREAS USING SUBPROBLEMS

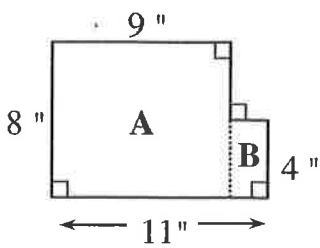
Students can use their knowledge of areas of polygons to find the areas of more complicated figures. The use of subproblems (that is, solving smaller problems in order to solve a larger problem) is one way to find the areas of complicated figures.

### Example 1

Find the area of the figure at right.



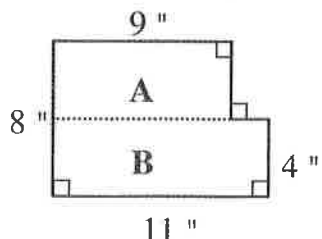
Method #1



Subproblems:

1. Find the area of rectangle A:  
 $8 \cdot 9 = 72$  square inches
2. Find the area of rectangle B:  
 $4 \cdot (11 - 9) = 4 \cdot 2 = 8$  square inches
3. Add the area of rectangle A to the area of rectangle B:  
 $72 + 8 = 80$  square inches

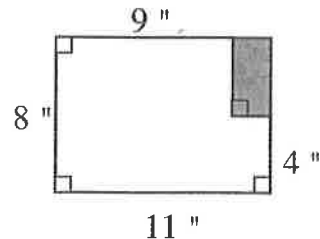
Method #2



Subproblems:

1. Find the area of rectangle A:  
 $9 \cdot (8 - 4) = 9 \cdot 4 = 36$  square inches
2. Find the area of rectangle B:  
 $11 \cdot 4 = 44$  square inches
3. Add the area of rectangle A to the area of rectangle B:  
 $36 + 44 = 80$  square inches

Method #3

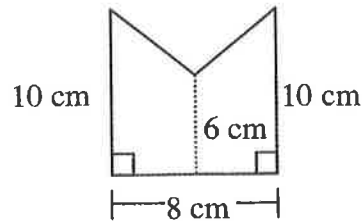


Subproblems:

1. Make a large rectangle by enclosing the upper right corner.
2. Find the area of the new, larger rectangle:  
 $8 \cdot 11 = 88$  square inches
3. Find the area of the shaded rectangle:  
 $(8 - 4) \cdot (11 - 9)$   
 $= 4 \cdot 2 = 8$  square inches
4. Subtract the shaded rectangle from the larger rectangle:  
 $88 - 8 = 80$  square inches

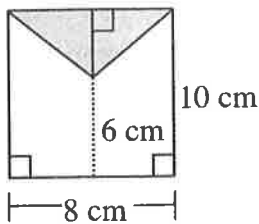
## Example 2

Find the area of the figure at right.



Subproblems:

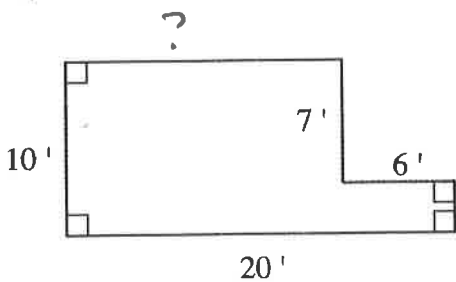
1. Make a rectangle out of the figure by enclosing the top.
2. Find the area of the entire rectangle:  $8 \cdot 10 = 80$  square cm
3. Find the area of the shaded triangle. Use the formula  $A = \frac{1}{2}bh$ .  $b = 8$  and  $h = 10 - 6 = 4$ , so  $A = \frac{1}{2}(8 \cdot 4) = \frac{32}{2} = 16$  square cm.
4. Subtract the area of the triangle from the area of the rectangle:  $80 - 16 = 64$  square cm.



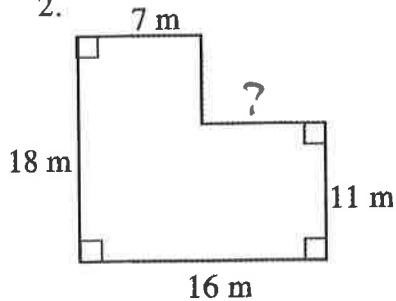
## Problems

Find the areas of the figures below.

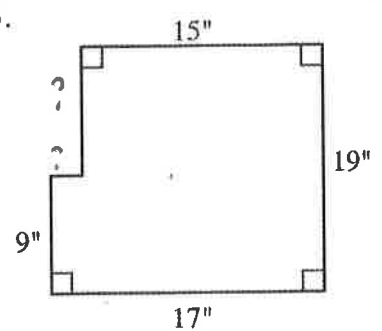
1.



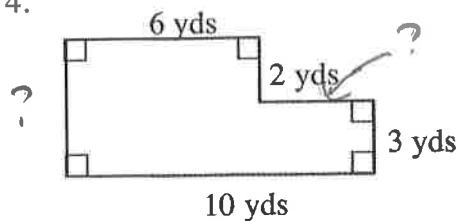
2.



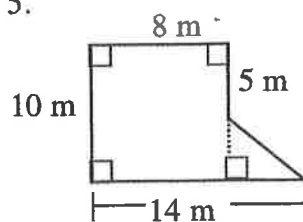
3.



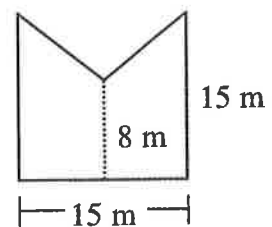
4.



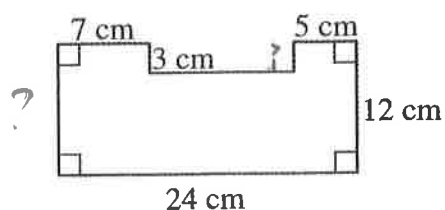
5.



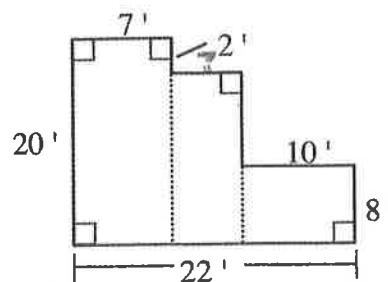
6.



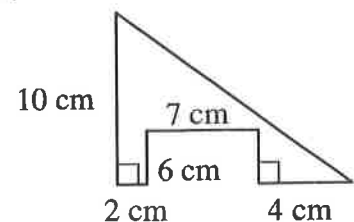
7.



8.



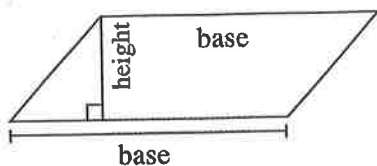
9.



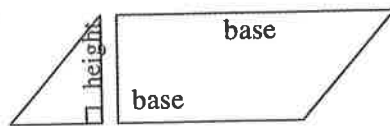
I

## AREA OF A PARALLELOGRAM

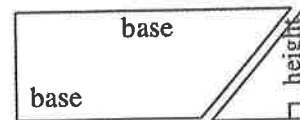
A parallelogram is easily changed to a rectangle by separating a triangle from one end of the parallelogram and moving it to the other end as shown in the three figures below. For additional information, see the Math Notes box in Lesson 5.3.3 of the *Core Connections, Course 1* text.



parallelogram  
Step 1



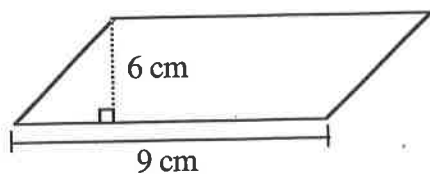
move triangle  
Step 2



rectangle  
Step 3

To find the area of a parallelogram, multiply the base times the height as you did with the rectangle:  $A = bh$ .

### Example



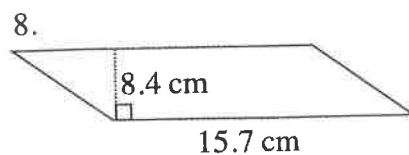
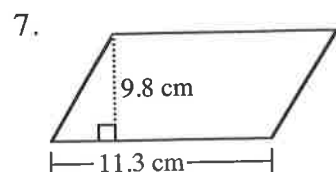
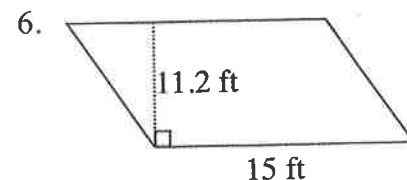
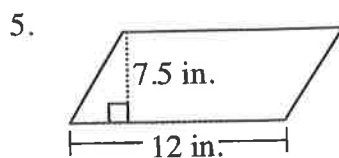
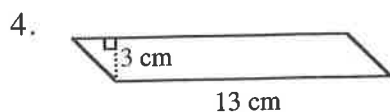
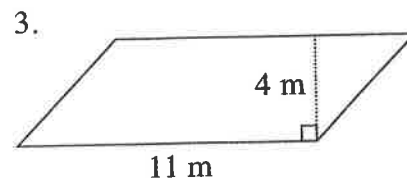
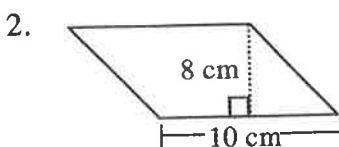
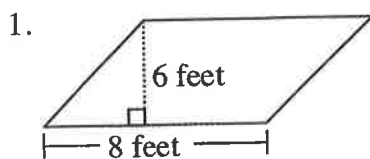
base = 9 cm

height = 6 cm

$A = 9 \cdot 6 = 54$  square cm

### Problems

Find the area of each parallelogram below.

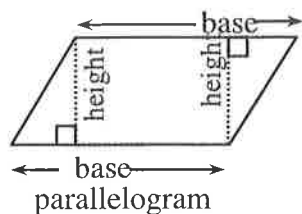


## Answers

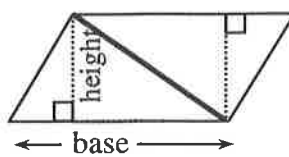
- |                |               |                  |                  |
|----------------|---------------|------------------|------------------|
| 1. 48 sq. feet | 2. 80 sq. cm  | 3. 44 sq. m      | 4. 39 sq. cm     |
| 5. 90 sq. in.  | 6. 168 sq. ft | 7. 110.74 sq. cm | 8. 131.88 sq. cm |

## AREA OF A TRIANGLE

The area of a triangle is equal to one-half the area of a parallelogram. This fact can easily be shown by cutting a parallelogram in half along a diagonal (see below). For additional information, see Math Notes box in Lesson 5.3.4 of the *Core Connections, Course 1* text.

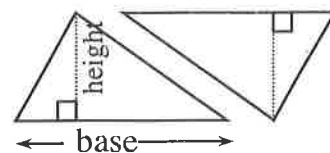


Step 1



draw a diagonal

Step 2



match triangles by cutting apart  
or by folding

Step 3

As you match the triangles by either cutting the parallelogram apart or by folding along the diagonal, the result is two congruent (same size and shape) triangles. Thus, the area of a triangle has half the area of the parallelogram that can be created from two copies of the triangle.

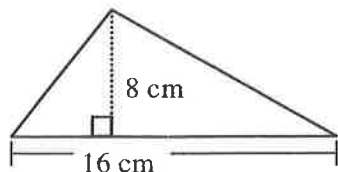
To find the area of a triangle, follow the steps below.

1. Identify the base.
2. Identify the height.
3. Multiply the base times the height.
4. Divide the product of the base times the height by 2:  $A = \frac{bh}{2}$  or  $\frac{1}{2}bh$ .

### Example 1

base = 16 cm

height = 8 cm

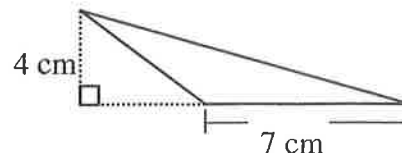


$$A = \frac{16 \cdot 8}{2} = \frac{128}{2} = 64\text{cm}^2$$

### Example 2

base = 7 cm

height = 4 cm

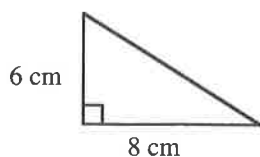


$$A = \frac{7 \cdot 4}{2} = \frac{28}{2} = 14\text{cm}^2$$

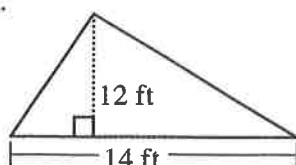


**Problems**

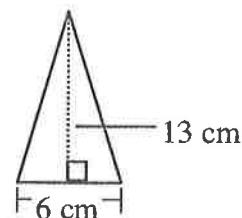
1.



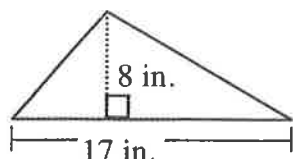
2.



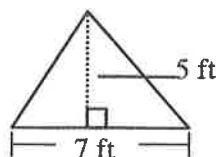
3.



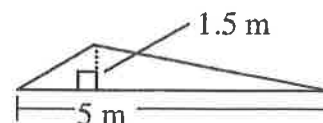
4.



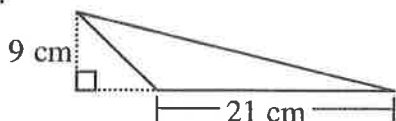
5.



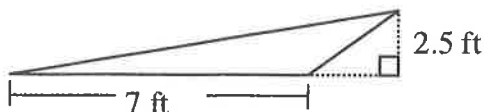
6.



7.



8.

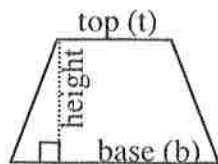


**Answers**

1. 24 sq. cm      2. 84 sq. ft      3. 39 sq. cm      4. 68 sq. in.  
 5. 17.5 sq. ft      6. 3.75 sq. m      7. 94.5 sq. cm      8. 8.75 sq. ft

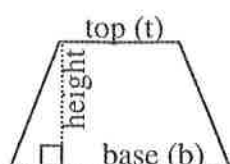
**AREA OF A TRAPEZOID**

A trapezoid is another shape that can be transformed into a parallelogram. Change a trapezoid into a parallelogram by following the three steps below.



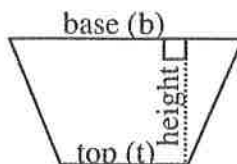
Trapezoid

Step 1



duplicate the trapezoid and rotate

Step 2



put the two trapezoids together to form a parallelogram

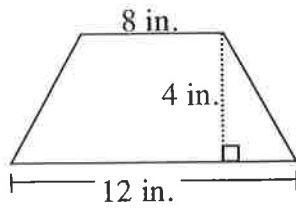
Step 3

To find the area of a trapezoid, multiply the base of the large parallelogram in Step 3 (base and top) times the height and then take half of the total area. Remember to add the lengths of the base and the top of the trapezoid before multiplying by the height. Note that some texts call the top length the upper base and the base the lower base.

$$A = \frac{1}{2}(b+t)h \quad \text{or} \quad A = \frac{b+t}{2} \cdot h$$

For additional information, see the Math Notes box in Lesson 6.1.1 of the *Core Connections, Course 1* text.

### Example



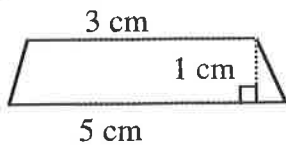
top = 8 in.  
base = 12 in.  
height = 4 in.

$$A = \frac{8+12}{2} \cdot 4 = \frac{20}{2} \cdot 4 = 10 \cdot 4 = 40 \text{ in.}^2$$

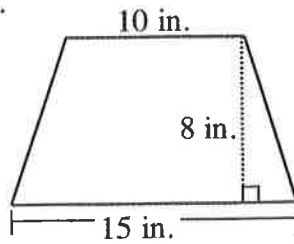
### Problems

Find the areas of the trapezoids below.

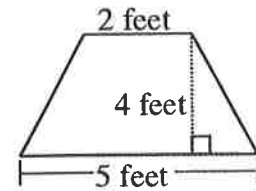
1.



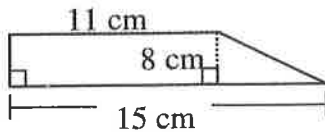
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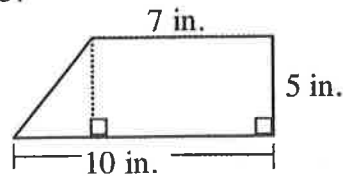
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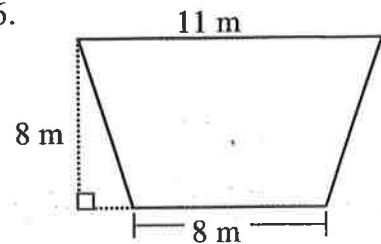
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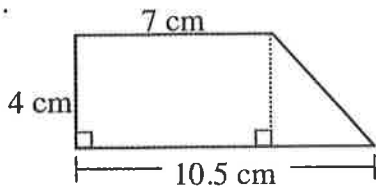
5.



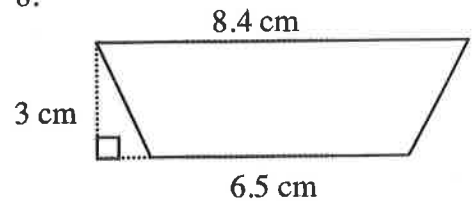
6.



7.



8.



### Answers

1. 4 sq. cm

2. 100 sq. in.

3. 14 sq. feet

4. 104 sq. cm

5. 42.5 sq. in.

6. 76 sq. m

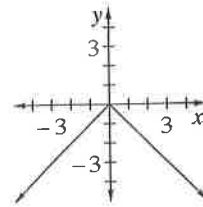
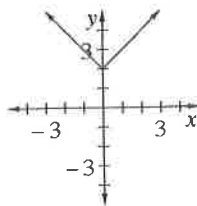
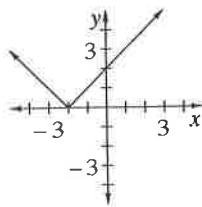
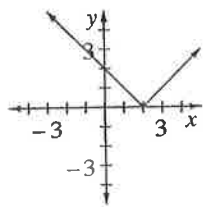
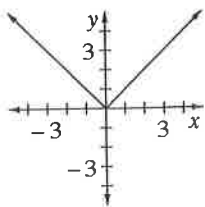
7. 35 sq. cm

8. 22.35 sq. cm.

J

**Answers**

- |            |       |                |            |            |
|------------|-------|----------------|------------|------------|
| 1. 11      | 2. 12 | 3. 4, -4       | 4. 16, -16 | 5. 24, -24 |
| 6. 13, -13 | 7. 9  | 8. no solution | 9. 13, -13 | 10. -7     |
| 11. 7, -7  | 12. 7 | 13. 3          | 14. 9      | 15. 3      |
| 16.        | 17.   | 18.            | 19.        | 20.        |



**ORDER OF OPERATIONS**

When students are first given expressions like  $3 + 4 \cdot 2$ , some students think the answer is 14 and some think the answer is 11. This is why mathematicians decided on a method to simplify an expression that uses more than one operation so that everyone can agree on the answer.

There is a set of rules to follow that provides a consistent way for everyone to evaluate expressions. These rules, called the Order of Operations, must be followed in order to arrive at a correct answer. As indicated by the name, these rules state the order in which the mathematical operations are to be completed.

For additional information, see the Math Notes boxes in Lesson 6.2.1 of the *Core Connections, Course 1* text, Lesson 3.1.2 of the *Core Connections, Course 2* text, or Lesson 2.1.4 of the *Core Connections, Course 3* text. For additional examples and practice, see the *Core Connections, Course 2* Checkpoint 5 materials or the *Core Connections, Course 3* Checkpoint 2 materials.

The first step is to organize the numerical expression into parts called terms, which are single numbers or products of numbers. A numerical expression is made up of a sum or difference of terms.

Examples of numerical terms are: 4,  $3(6)$ ,  $6(9 - 4)$ ,  $2 \cdot 3^2$ ,  $3(5 + 2^3)$ , and  $\frac{16-4}{6}$ .

For the problem above,  $3 + 4 \cdot 2$ , the terms are circled at right.  $(3) + (4 \cdot 2)$

Each term is simplified separately, giving  $3 + 8$ . Then the terms are added:  $3 + 8 = 11$ . Thus,  $3 + 4 \cdot 2 = 11$ .

**Example 1**

To evaluate an expression:

- Circle each term in the expression.
- Simplify each term until it is one number by:
  - Simplifying the expressions within the parentheses.
  - Evaluating each exponential part (e.g.,  $3^2$ ).
  - Multiplying and dividing from left to right.
- Finally, combine terms by adding or subtracting from left to right.

$$2 \cdot 3^2 + 3(6 - 3) + 10$$

$$(2 \cdot 3^2) + (3(6 - 3)) + (10)$$

$$(2 \cdot 3^2) + (3(3)) + (10)$$

$$(2 \cdot 9) + (3(3)) + (10)$$

$$(18) + (9) + (10)$$

$$27 + 10$$

$$37$$

**Example 2**

- Circle the terms.
- Simplify inside the parentheses.
- Simplify the exponents.
- Multiply and divide from left to right.

Finally, add and subtract from left to right.

$$5 - 8 \div 2^2 + 6(5 + 4) - 5^2$$

$$a. (5) - (8 \div 2^2) + (6(5 + 4)) - (5^2)$$

$$b. (5) - (8 \div 2^2) + (6(9)) - (5^2)$$

$$c. (5) - (8 \div 4) + (6(9)) - (25)$$

$$d. 5 - 2 + 54 - 25$$

$$32$$

**Example 3**

- Circle the terms.
- Multiply and divide left to right, including exponents.

Add or subtract from left to right.

$$20 + \frac{5+7}{3} - 4^2 + 12 \div 4$$

$$a. (20) + (\frac{5+7}{3}) - (4^2) + (12 \div 4)$$

$$b. 20 + 4 - 16 + 3$$

$$11$$

## Problems

Circle the terms, then simplify each expression.

1.  $5 \cdot 3 + 4$

2.  $10 \div 5 + 3$

3.  $2(9 - 4) \cdot 7$

4.  $6(7 + 3) + 8 \div 2$

5.  $15 \div 3 + 7(8 + 1) - 6$

6.  $\frac{9}{3} + 5 \cdot 3^2 - 2(14 - 5)$

7.  $\frac{20}{6+4} + 7 \cdot 2 \div 2$

8.  $\frac{5+30}{7} + 6^2 - 18 \div 9$

9.  $2^3 + 8 - 16 \div 8 \cdot 2$

10.  $25 - 5^2 + 9 - 3^2$

11.  $5(17 - 7) + 4 \cdot 3 - 8$

12.  $(5 - 2)^2 + (9 + 1)^2$

13.  $4^2 + 9(2) \div 6 + (6 - 1)^2$

14.  $\frac{18}{3^2} + \frac{5 \cdot 3}{5}$

15.  $3(7 - 2)^2 + 8 \div 4 - 6 \cdot 5$

16.  $14 \div 2 + 6 \cdot 8 \div 2 - (9 - 3)^2$

17.  $\frac{27}{3} + 18 - 9 \div 3 - (3 + 4)^2$

18.  $26 \cdot 2 \div 4 - (6 + 4)^2 + 3(5 - 2)^3$

19.  $\left(\frac{42+3}{5}\right)^2 + 3^2 - (5 \cdot 2)^2$

## Answers

1. 19

2. 5

3. 70

4. 64

5. 62

6. 30

7. 9

8. 39

9. 12

10. 0

11. 54

12. 109

13. 44

14. 5

15. 47

16. -5

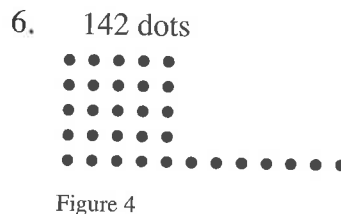
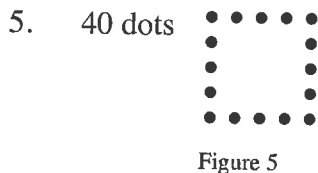
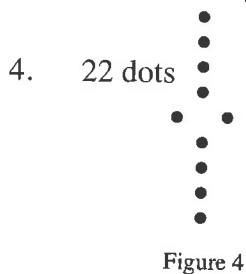
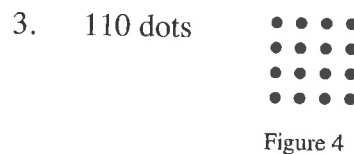
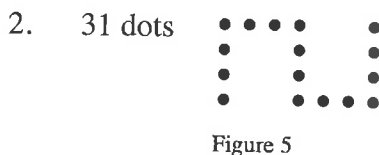
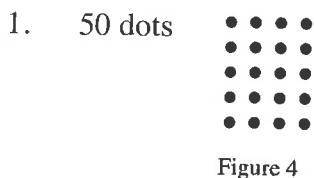
17. -25

18. -6

19. -10

K

Answers



**DISTRIBUTIVE PROPERTY**

The Distributive Property shows how to express sums and products in two ways:  
 $a(b + c) = ab + ac$ . This can also be written  $(b + c)a = ab + ac$ .

Factored form  
 $a(b + c)$

Distributed form  
 $a(b) + a(c)$

Simplified form  
 $ab + ac$

To simplify: Multiply each term on the inside of the parentheses by the term on the outside.  
 Combine terms if possible.

For additional information, see the Math Notes boxes in Lessons 2.3.4 and 7.3.2 of the *Core Connections, Course 1* text, Lesson 4.3.3 of the *Core Connections, Course 2* text, or Lesson 3.2.5 of the *Core Connections, Course 3* text. For additional examples and practice, see the *Core Connections, Course 1* Checkpoint 8A materials.

**Example 1**

$$\begin{aligned} 2(47) &= 2(40 + 7) \\ &= (2 \cdot 40) + (2 \cdot 7) \\ &= 80 + 14 = 94 \end{aligned}$$

**Example 2**

$$\begin{aligned} 3(x + 4) &= (3 \cdot x) + (3 \cdot 4) \\ &= 3x + 12 \end{aligned}$$

**Example 3**

$$\begin{aligned} 4(x + 3y + 1) &= (4 \cdot x) + (4 \cdot 3y) + 4(1) \\ &= 4x + 12y + 4 \end{aligned}$$

## Problems

Simplify each expression below by applying the Distributive Property.

- |                        |                        |                 |                 |
|------------------------|------------------------|-----------------|-----------------|
| 1. $6(9 + 4)$          | 2. $4(9 + 8)$          | 3. $7(8 + 6)$   | 4. $5(7 + 4)$   |
| 5. $3(27) = 3(20 + 7)$ | 6. $6(46) = 6(40 + 6)$ | 7. $8(43)$      | 8. $6(78)$      |
| 9. $3(x + 6)$          | 10. $5(x + 7)$         | 11. $8(x - 4)$  | 12. $6(x - 10)$ |
| 13. $(8 + x)4$         | 14. $(2 + x)5$         | 15. $-7(x + 1)$ | 16. $-4(y + 3)$ |
| 17. $-3(y - 5)$        | 18. $-5(b - 4)$        | 19. $-(x + 6)$  | 20. $-(x + 7)$  |
| 21. $-(x - 4)$         | 22. $-(-x - 3)$        | 23. $x(x + 3)$  | 24. $4x(x + 2)$ |
| 25. $-x(5x - 7)$       | 26. $-x(2x - 6)$       |                 |                 |

## Answers

- |   |   |                   |                     |
|---|---|-------------------|---------------------|
| 1. $(6 \cdot 9) + (6 \cdot 4) = 54 + 24 = 78$ | 2. $(4 \cdot 9) + (4 \cdot 8) = 36 + 32 = 68$ |                   |                     |
| 3. $56 + 42 = 98$                             | 4. $35 + 20 = 55$                             | 5. $60 + 21 = 81$ | 6. $240 + 36 = 276$ |
| 7. $320 + 24 = 344$                           | 8. $420 + 48 = 468$                           | 9. $3x + 18$      | 10. $5x + 35$       |
| 11. $8x - 32$                                 | 12. $6x - 60$                                 | 13. $4x + 32$     | 14. $5x + 10$       |
| 15. $-7x - 7$                                 | 16. $-4y - 12$                                | 17. $-3y + 15$    | 18. $-5b + 20$      |
| 19. $-x - 6$                                  | 20. $-x - 7$                                  | 21. $-x + 4$      | 22. $x + 3$         |
| 23. $x^2 + 3x$                                | 24. $4x^2 + 8x$                               | 25. $-5x^2 + 7x$  | 26. $-2x^2 + 6x$    |

When the Distributive Property is used to reverse, it is called factoring. Factoring changes a sum of terms (no parentheses) to a product (with parentheses.)

$$ab + ac = a(b + c)$$

To factor: Write the common factor of all the terms outside of the parentheses. Place the remaining factors of each of the original terms inside of the parentheses.

For additional examples and practice, see the *Core Connections, Course 1* Checkpoint 8A materials.

**Example 4**

$$\begin{aligned} 4x + 8 &= 4 \cdot x + 4 \cdot 2 \\ &= 4(x + 2) \end{aligned}$$

**Example 5**

$$\begin{aligned} 6x^2 - 9x &= 3x \cdot 2x - 3x \cdot 3 \\ &= 3x(2x - 3) \end{aligned}$$

**Example 6**

$$\begin{aligned} 6x + 12y + 3 &= 3 \cdot 2x + 3 \cdot 4y + 3 \cdot 1 \\ &= 3(2x + 4y + 1) \end{aligned}$$

**Problems**

Factor each expression below by using the Distributive Property in reverse.

- |                    |                    |                        |                    |
|--------------------|--------------------|------------------------|--------------------|
| 1. $6x + 12$       | 2. $5y - 10$       | 3. $8x + 20z$          | 4. $x^2 + xy$      |
| 5. $8m + 24$       | 6. $16y + 40$      | 7. $8m - 2$            | 8. $25y - 10$      |
| 9. $2x^2 - 10x$    | 10. $21x^2 - 63$   | 11. $21x^2 - 63x$      | 12. $15y + 35$     |
| 13. $4x + 4y + 4z$ | 14. $6x + 12y + 6$ | 15. $14x^2 - 49x + 28$ | 16. $x^2 - x + xy$ |

**Answers**

- |                    |                     |                        |                    |
|--------------------|---------------------|------------------------|--------------------|
| 1. $6(x + 2)$      | 2. $5(y - 2)$       | 3. $4(2x + 5z)$        | 4. $x(x + y)$      |
| 5. $8(m + 3)$      | 6. $8(2y + 5)$      | 7. $4(2m - 1)$         | 8. $5(5y - 2)$     |
| 9. $2x(x - 5)$     | 10. $21(x^2 - 3)$   | 11. $21x(x - 3)$       | 12. $5(3y + 7)$    |
| 13. $4(x + y + z)$ | 14. $6(x + 2y + 1)$ | 15. $7(2x^2 - 7x + 4)$ | 16. $x(x - 1 + y)$ |



## VARIABLE EXPRESSIONS

A variable is a symbol used to represent one or more numbers. It is common to use letters of the alphabet for variables. The value of a variable used several times in one expression must be the same. For additional practice and more examples, see the *Core Connections, Course 2* Checkpoint 6 materials.

### Example 1

If the unknown distance of Cecil's hop is represented by the variable  $h$ , write an expression for:

- Three equal hops  $\Rightarrow h+h+h$  or  $3h$
- Five equal hops  $\Rightarrow h+h+h+h+h$  or  $5h$
- Two equal hops and walking 3 feet  $\Rightarrow h+h+3$  or  $2h+3$

### Example 2

If the unknown cost of a banana is  $b$ , and the unknown cost of an apple is  $a$ , write an expression for the cost of:

- Three bananas and two apples  $\Rightarrow b+b+b+a+a$  or  $3b+2a$
- One banana and 3 apples  $\Rightarrow b+a+a+a$  or  $b+3a$
- One banana, one apple, a \$2 item, and a \$3 item  $\Rightarrow b+a+2+3$  or  $b+a+5$

**Problems**

If the unknown distance of Cecil's jump is represented by  $J$ , write an expression for:

1. Three jumps
2. Six jumps
3. Four jumps and walking 5 feet
4. Walking 3 feet, two jumps, walking 2 feet

If the unknown distance of Cecil's jump is represented by  $J$ , and the unknown distance of Cecil's hop is represented by  $H$ , write an expression for:

5. Two jumps and two hops
6. One jump, three hops, and two jumps
7. One jump, three hops and walking 7 feet
8. Walking 6 feet, three hops, and two jumps

If the unknown cost of a taco is  $T$ , and the unknown cost of a carton of milk is  $M$ , write an expression for the cost of:

9. Three tacos and two milks
10. One taco and four milks
11. One taco, one milk and two tacos, one milk
12. Two tacos, one milk, and a \$2 item

**Answers**

1.  $J + J + J = 3J$
2.  $J + J + J + J + J + J = 6J$
3.  $J + J + J + J + 5 = 4J + 5$
4.  $3 + J + J + 2 = 2J + 5$
5.  $J + J + H + H = 2J + 2H$
6.  $J + H + H + H + J + J = 3J + 3H$
7.  $J + H + H + H + 7 = J + 3H + 7$
8.  $6 + H + H + H + J + J = 3H + 2J + 6$
9.  $3T + 2M$
10.  $T + 4M$
11.  $T + M + 2T + M = 3T + 2M$
12.  $2T + M + 2$

## SUBSTITUTION AND EVALUATION OF EXPRESSIONS

Substitution is replacing one symbol with an equivalent symbol (a number, a variable, or an expression). One application of the substitution property is replacing a variable name with a number in an expression or equation. A variable is a letter used to represent one or more numbers (or other algebraic expression). The numbers are the values of the variable. A variable expression has numbers and variables with arithmetic operations performed on it.

In general, if  $a = b$ , then  $a$  may replace  $b$  and  $b$  may replace  $a$ .

After numerical substitutions have been made, following the order of operations and doing the arithmetic will correctly evaluate the expression.

For additional information, see the Math Notes box in Lesson 4.2.2 of the *Core Connections, Course 1* text, Lesson 3.2.1 of the *Core Connections, Course 2* text, or Lesson 2.1.4 of the *Core Connections, Course 3* text. For additional examples and practice, see the *Core Connections, Course 1* Checkpoint 8A materials or the *Core Connections, Course 2* Checkpoint 6 materials.

### Examples

Evaluate each variable expression for  $x = 3$ .

a.  $5x \Rightarrow 5(3) \Rightarrow 15$

b.  $x + 10 \Rightarrow (3) + 10 \Rightarrow 13$

c.  $\frac{18}{x} \Rightarrow \frac{18}{3} \Rightarrow 6$

d.  $\frac{x}{3} \Rightarrow \frac{3}{3} \Rightarrow 1$

e.  $3x - 5 \Rightarrow 3(3) - 5 \Rightarrow 9 - 5 \Rightarrow 4$

f.  $5x + 3x \Rightarrow 5(3) + 3(3) \Rightarrow 15 + 9 \Rightarrow 24$

### Problems

Evaluate each of the variable expressions below for  $x = -4$  and  $y = 3$ . Be sure to follow the Order of Operations as you simplify each expression.

1.  $x + 4$

2.  $x - 1$

3.  $x + y + 3$

4.  $y - 3 + x$

5.  $x^2 - 5$

6.  $-x^2 + 5$

7.  $x^2 + 4x - 3$

8.  $-3x^2 + 2x$

9.  $x + 3 + 2y$

10.  $y^2 + 3x - 2$

11.  $x^2 + y^2 + 3^2$

12.  $2^2 + y^2 - 2x^2$

Evaluate the expressions below using the values of the variables in each problem. These problems ask you to evaluate each expression twice, once with each of the values.

13.  $3x^2 - 2x + 5$  for  $x = -3$  and  $x = 4$

14.  $2x^2 - 3x + 6$  for  $x = -2$  and  $x = 5$

15.  $-3x^2 + 7$  for  $x = -3$  and  $x = 2$

16.  $-2x^2 + 5$  for  $x = -4$  and  $x = 5$

Evaluate the variable expressions for  $x = -4$  and  $y = 3$ .

17.  $x(x + 3x)$

18.  $2(x + 2x)$

19.  $2(x + y) + 4\left(\frac{y+2}{x}\right)$

20.  $3\left(y^2 + 2\left(\frac{x+7}{2}\right)\right)$

21.  $2y(x + x^2 - 2y)$

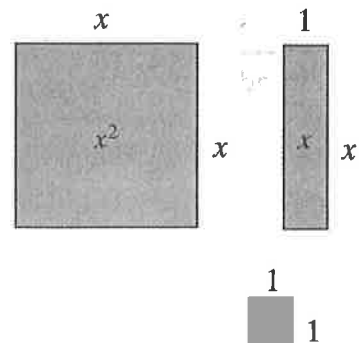
22.  $(3x + y)(2x + 4y)$

**Answers**

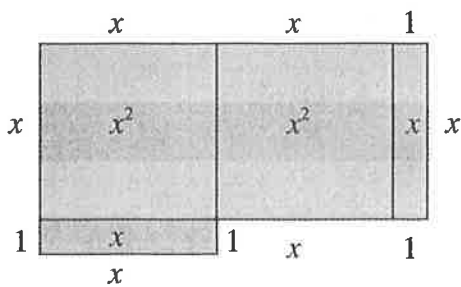
- |              |         |            |            |             |
|--------------|---------|------------|------------|-------------|
| 1. 0         | 2. -5   | 3. 2       | 4. -4      | 5. 11       |
| 6. -11       | 7. -3   | 8. -56     | 9. 5       | 10. -5      |
| 11. 34       | 12. -19 | 13. 38; 45 | 14. 20; 41 | 15. -20; -5 |
| 16. -27; -45 | 17. 36  | 18. -18    | 19. 20     | 20. 36      |
| 21. -16      | 22. -50 |            |            |             |

**ALGEBRA TILES AND PERIMETER**

Algebraic expressions can be represented by the perimeters of algebra tiles (rectangles and squares) and combinations of algebra tiles. The dimensions of each tile are shown along its sides and the tile is named by its area as shown on the tile itself in the figures at right. When using the tiles, perimeter is the distance around the exterior of a figure. For additional information, see the Math Notes box in Lesson 6.2.4 of the *Core Connections, Course 1* text.

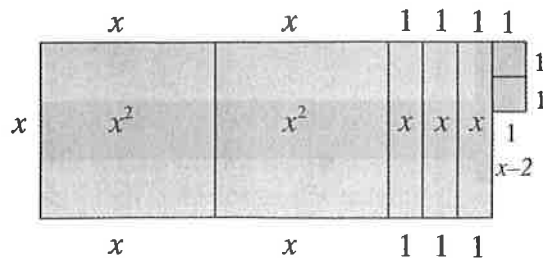


**Example 1**



$P = 6x + 4$  units

**Example 2**



$P = 6x + 8$  units

**Problems**

## MEASURES OF CENTRAL TENDENCY

Measures of central tendency are numbers that locate or approximate the “center” of a set of data—that is, a “typical” value that describes the set of data. Mean and median are the most common measures of central tendency. (Mode will not be covered in this course.)

The mean is the arithmetic average of a data set. Add all the values in a set and divide this sum by the number of values in the set. The median is the middle number in a set of data arranged numerically. An outlier is a number that is much smaller or larger than most of the others in the data set. The range of a data set is the difference between the highest and lowest values of the data set.

For additional information, see the Math Notes boxes in Lesson 8.1.2 of the *Core Connections, Course 1* text or Lessons 1.1.3 and 1.1.4 of the *Core Connections, Course 2* text.

The mean is calculated by finding the sum of the data set and dividing it by the number of elements in the set.

### Example 1

Find the mean of this set of data: 34, 31, 37, 44, 38, 34, 42, 34, 43, and 41.

- $34 + 31 + 37 + 44 + 38 + 34 + 42 + 34 + 43 + 41 = 378$
- $\frac{378}{10} = 37.8$

The mean of this set of data is 37.8.

### Example 2

Find the mean of this set of data: 92, 82, 80, 92, 78, 75, 95, and 77.

- $92 + 82 + 80 + 92 + 78 + 75 + 95 + 77 + 77 = 748$
- $\frac{748}{9} = 83.1$

The mean of this set of data is 83.1.

### Problems

Find the mean of each set of data.

1. 29, 28, 34, 30, 33, 26, and 34.
2. 25, 34, 35, 27, 31, and 30.
3. 80, 89, 79, 84, 95, 79, 78, 89, 76, 82, 76, 92, 89, 81, and 123.
4. 116, 104, 101, 111, 100, 107, 113, 118, 113, 101, 108, 109, 105, 103, and 91.

The median is the middle number in a set of data *arranged in numerical order*. If there is an even number of values, the median is the mean (average) of the two middle numbers.

### Example 3

Find the median of this set of data: 34, 31, 37, 44, 38, 34, 43, and 41.

- Arrange the data in order: 31, 34, 34, 34, 37, 38, 41, 43, 44.
- Find the middle value(s): 37 and 38.
- Since there are two middle values, find their mean:  $37 + 38 = 75$ ,  $\frac{75}{2} = 37.5$ . Therefore, the median of this data set is 37.5.

### Example 4

Find the median of this set of data: 92, 82, 80, 92, 78, 75, 95, 77, and 77.

- Arrange the data in order: 75, 77, 77, 78, 80, 82, 92, 92, and 95.
- Find the middle value(s): 80. Therefore, the median of this data set is 80.

### Problems

Find median of each set of data.

5. 29, 28, 34, 30, 33, 26, and 34.

6. 25, 34, 27, 25, 31, and 30.

7. 80, 89, 79, 84, 95, 79, 78, 89, 76, 82, 76, 92, 89, 81, and 123.

8. 116, 104, 101, 111, 100, 107, 113, 118, 113, 101, 108, 109, 105, 103, and 91.

The range of a set of data is the difference between the highest value and the lowest value.

### Example 5

Find the range of this set of data: 114, 109, 131, 96, 140, and 128.

- The highest value is 140.
- The lowest value is 96.
- $140 - 96 = 44$ .
- The range of this set of data is 44.

### Example 6

Find the range of this set of data: 37, 44, 36, 29, 78, 15, 57, 54, 63, 27, and 48.

- The highest value is 78.
- The lowest value is 27.
- $78 - 27 = 51$ .
- The range of this set of data is 51.

## Problems

Find the range of each set of data in problems 5 through 8.

Outliers are numbers in a data set that are either much higher or much lower than the other numbers in the set.

### Example 7

Find the outlier of this set of data: 88, 90, 96, 93, 87, 12, 85, and 94.

- The outlier is 12.

### Example 8

Find the outlier of this set of data: 67, 54, 49, 76, 64, 59, 60, 72, 123, 44, and 66.

- The outlier is 123.

## Problems

Identify the outlier in each set of data.

9. 70, 77, 75, 68, 98, 70, 72, and 71.
10. 14, 22, 17, 61, 20, 16, and 15.
11. 1376, 1645, 1783, 1455, 3754, 1790, 1384, 1643, 1492, and 1776.
12. 62, 65, 93, 51, 55, 14, 79, 85, 55, 72, 78, 83, 91, and 76.

## Answers

- |                          |                            |                           |                            |
|--------------------------|----------------------------|---------------------------|----------------------------|
| 1. 30.57                 | 2. $30.\bar{3}$            | 3. $86.\bar{13}$          | 4. $106.\bar{6}$           |
| 5. median 30;<br>range 8 | 6. median 28.5;<br>range 9 | 7. median 82;<br>range 47 | 8. median 107;<br>range 27 |
| 9. 98                    | 10. 61                     | 11. 3754                  | 12. 14                     |

## GRAPHICAL REPRESENTATIONS OF DATA

Students represent distributions of single-variable data numerical data using dot plots, stem-and-leaf plots, box plots, and histograms. They represent categorical one-variable data on bar graphs. Each representation communicates information in a slightly different way.

### STEM-AND-LEAF-PLOTS

A stem-and-leaf plot is a way to display data that shows the individual values from a set of data and how the values are distributed. The “stem” part on the graph represents all of the digits except the last one. The “leaf” part of the graph represents the last digit of each number.

Read more about stem-and-leaf plots, and how they compare to dot plots and histograms, in the Math Notes in Lesson 2.1.2 and 2.2.1 in the *Core Connections, Course 1* text, and in the Math Notes in Lesson 7.1.1 of the *Core Connections, Course 2* text.

#### Example 1

Make a stem-and-leaf plot of this set of data: 34, 31, 37, 44, 38, 29, 34, 42, 43, 34, 52, and 41.

2	9
3	1 4 4 4 7 8
4	1 2 3 4
5	2

#### Example 2

Make a stem-and-leaf plot of this set of data: 392, 382, 380, 392, 378, 375, 395, 377, and 377.

37	5 7 7 8
38	0 2
39	2 2 5

#### Problems

Make a stem-and-leaf plot of each set of data.

- |  |   |
|--|---|
| <p>1. 29, 28, 34, 30, 33, 26, 18, and 34.</p> <p>3. 80, 89, 79, 84, 95, 79, 89, 67, 82, 76, 92, 89, 81, and 123.</p> | <p>2. 25, 34, 27, 25, 19, 31, 42, and 30.</p> <p>4. 116, 104, 101, 111, 100, 107, 113, 118, 113, 101, 108, 109, 105, 103, and 91.</p> |
|--|---|



## HISTOGRAMS AND BAR GRAPHS

Histograms and bar graphs are visual ways to represent data. Both consist of vertical bars (called bins) with heights that represent the number of data points (called the frequency) in each bin.

Histograms are for displaying distributions of numerical data. In a histogram each bar represents the number of data elements within a certain range of values. All the bars touch each other. Values at the left side of a bin's range are included in that bin. Each range of values should have the same width.

Bar graphs are for displaying categorical data. In a bar graph each bar represents the number of data elements in a certain category. All the bars are the same width and are separated from each other.

For additional information and examples, see the Math Notes boxes in Lessons 2.1.2 and 2.2.1 of the *Core Connections, Course 1* text, or Lesson 7.1.1 of the *Core Connections, Course 2* text. For additional examples and practice, see the *Core Connections, Course 1* Checkpoint 9A materials or *Connections, Course 2* Checkpoint 7B materials in the back of those texts.

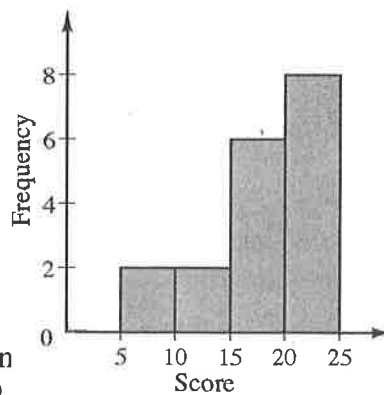
### Example 3

The scores for a 25-point quiz are listed below arranged from least to greatest.

7, 7, 12, 13, 15, 16, 16, 16, 18, 19, 20, 20, 20, 21, 21, 22, 23, 24

Using intervals of five points, create a histogram for the class.

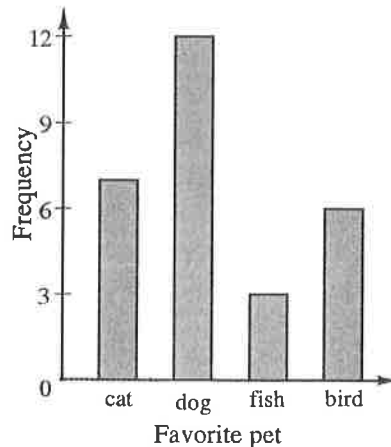
See histogram at right. Scores on the right end of the interval are included in the next interval. The interval between 10 and 15 only includes the two scores of 12 and 13. The interval between 15 and 20 only includes the six scores of 15, 16, 16, 16, 18, and 19.



### Example 4

Ms. Lim asked each of her students about their favorite kind of pet. Based on their responses, she drew the bar graph at right. Use the bar graph to answer each question.

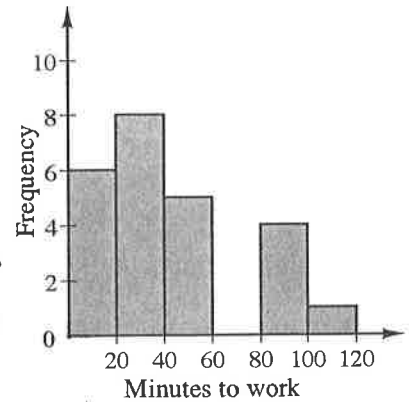
- What is the favorite pet?
- How many students chose a bird as their favorite pet?
- What was the least favorite pet?
- If every student voted once, how many students are in the class?



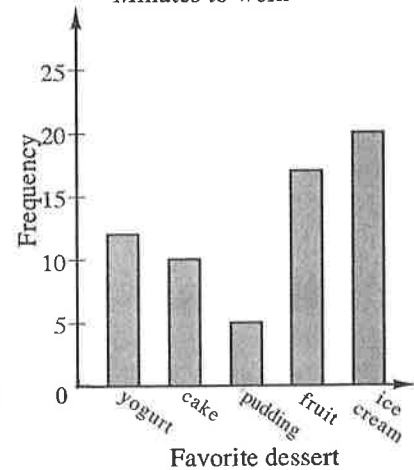
Answers: a. dog b. 6 c. fish d. 28

**Problems**

5. Mr. Diaz surveyed his employees about the time it takes them to get to work. The results are shown in the histogram at right.
  - a. How many employees completed the survey?
  - b. How many employees get to work in less than 20 minutes?
  - c. How many employees get to work in less than 40 minutes?
  - d. How many employees take 60 minutes to get to work?



6. The two sixth grade classes at Vista Middle School voted for their favorite dessert. The results are shown in the bar graph at right for the five favorite choices.
  - a. What was the favorite dessert and how many students made that choice?
  - b. How many students selected cake as their favorite dessert?
  - c. How many students selected yogurt as their favorite?
  - d. How many more students selected ice cream than pudding?



7. Mr. Fernandez asked 30 people at work how many pets they owned. The results are shown at right. Make a histogram to display this data. Use intervals of one pet.
8. During the first week of school Ms. Chan asked her students to name the country where they were born. There were so many different countries she grouped them by continent:

0 pets	5 people
1 pet	8 people
2 pets	10 people
3 pets	3 people
4 pets	2 people
5 pets	1 person
9 pets	1 person

North America: 14 students, South America: 2 students, Europe: 3 students, Asia: 10 students, Africa: 1 student, Australia: 0 students.

Make a bar graph to display this information.

9. Three coins were tossed 20 times and the number of results that were “heads” each time is shown below:

1, 1, 2, 0, 2, 3, 1, 2, 1, 2, 2, 1, 3, 2, 0, 1, 2, 0, 2, 1

Make a histogram to show the results.

10. The physical education teacher at West Middle School asked the class about their favorite winter activity. Here were the results:

reading: 8 students, ice skating: 4 students, skiing: 6 student, snowboarding: 11 students, computer activities: 14 students.

Make a bar graph to show the results.