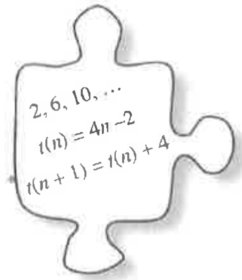
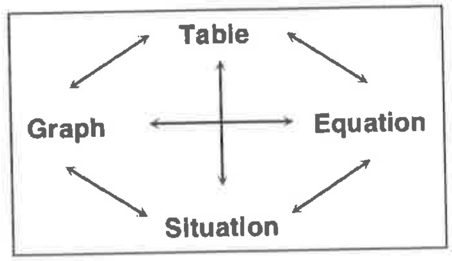
5.2.1 How can I describe a sequence?



Generating and Investigating Sequences

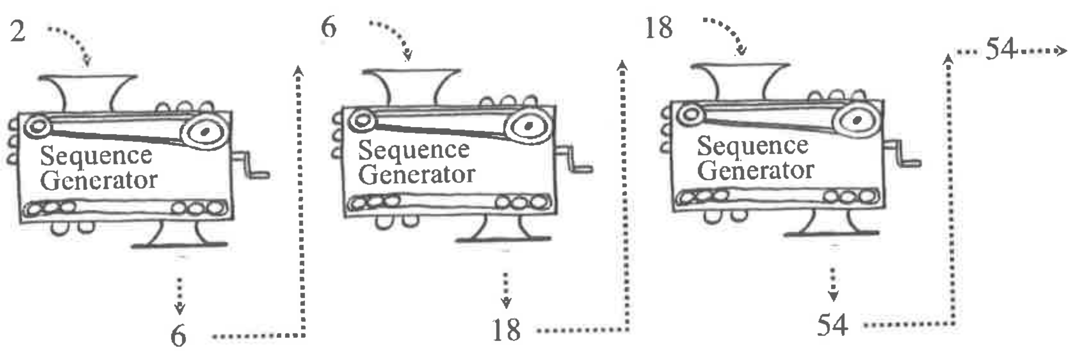
In the bouncing ball activity from Lessons 5.1.2 and 5.1.3, you used multiple representations (a table, an equation, and a graph) to represent a discrete situation involving a bouncing ball (a situation). Today you will learn about a new way to represent a discrete pattern, called a sequence.



5-40. Samantha was thinking about George and Lenny and their rabbits. When she listed the number of rabbits George and Lenny could have each month, she ended up with the ordered list below, called a **sequence**.

$$2, 6, 18, 54, \ldots$$

She realized that she could represent this situation using a sequence-generating machine that would generate the number of rabbits each month by doing something to the previous month's number of rabbits. She tested her generator by putting in a **first term** of 2 and she recorded each output before putting it into the next machine. Below is the diagram she used to explain her idea to her teammates.



a. What does Samantha's sequence generator seem to be doing to each input? **[Answers vary, but students are expected to notice that the inputs are multiplied by 3.]**

b. What are the next two terms of Samantha's sequence? Show how you got your answer. **[54(3) = 162, 162(3) = 486]**

c. Samantha decided to use the same sequence generator, but this time she started with a first term of 5. What are the next four terms in this new sequence? **[15, 45, 135, 405.]**

5-41. SEQUENCE FAMILIES

Samantha and her teacher have been busy creating new sequence generators and the sequences they produce. Below are the sequences Samantha and her teacher created.

a. $-4, -1, 2, 5, \ldots$

b. $1.5, 3, 6, 12, \ldots$

c. $0, 1, 4, 9, \ldots$

d. $2, 3.5, 5, 6.5, \ldots$

e. $1, 1, 2, 3, 5, 8, \ldots$

f. $9, 7, 5, 3, \ldots$

g. $48, 24, 12, \ldots$

h. $27, 9, 3, 1, \ldots$

i. $8, 2, 0, 2, 8, 18, \ldots$

j. $\frac{5}{4}, \frac{5}{2}, 5, 10, \ldots$

Your teacher will give your team a set of Lesson 5.2.1A Resource Pages with the above sequences on strips so that everyone in your team can see and work with them in the middle of your workspace.

Your Task: Working together, organize the sequences into families of similar sequences. Your team will need to decide how many families to make, what common features make the sequences a family, and what characteristics make each family different from the others. Follow the directions below. As you work, use the following questions to help guide your team's discussion.

Discussion Points

How can we describe the pattern?

How does it grow?

What do they have in common?

(1) As a team, initially sort the sequence strips into groups based on your first glance at the sequences. Remember that you can sort the sequences into more than two families. You will have a chance to revise your groups of sequences throughout this activity, so just sort them in a way that makes sense to start out with. Which seem to behave similarly? Record your groupings and what they have in common before proceeding. **[Answers vary.]**

Problem continues on next page. →

5-41. *Problem continued from previous page.*

(2) If one exists, find a sequence generator (growth pattern) for each sequence and write it on the strip. You can express the sequence generator either in symbols or in words. Also record the next three terms in each sequence on the strips. Do your sequence families still make sense? If so, what new information do you have about your sequence families? If not, reorganize the strips and explain how you decided to group them. **[a: 8, 11, 14, add 3; b: 24, 48, 96, multiply by 2; c: 16, 25, 36, next perfect square or add next odd; d: 8, 9.5, 11, add 1.5; e: 13, 21, 34, add previous two terms; f: 1, –1, –3, subtract 2; g: 6, 3, 1.5, divide by 2; h: $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, divide by 3; i: 32, 50, 72, add –6 then –2 then 2, then 6 and so on (the difference increases by 4 each time); j: 20, 40, 80, multiply by 2]**

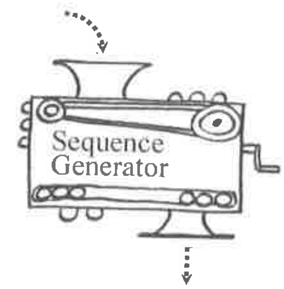
(3) Get a set of Lesson 5.2.1B Resource Pages for your team. Then record each sequence in a table. Your table should compare the **term number**, n, to the value of each **term**, $t(n)$. This means that your sequence itself is a list of *outputs* of the relationship and the *inputs* are a list of integers! The first term in a sequence is always $n = 1$. Attach the table to the sequence strip it represents. Do your sequence families still make sense? Record any new information or reorganize your sequence families if necessary.

(4) Now graph each sequence on a Lesson 5.2.1C Resource Page. Include as many terms as will fit on the existing set of axes. Be sure to decide whether your graphs should be discrete or continuous. Use color to show the growth between the points on each graph. Attach the graph to the sequence strip it represents. Do your sequence families still make sense? Record any new information and reorganize your sequence families if necessary. **[Answer graphs can be found on the Lesson 5.2.1D Resource Page.]**

5-42. Choose one of the families of sequences you created in problem 5-41. With your team, write clear summary statements about this family of sequences. Refer to the Discussion Points in problem 5-41 to help you write summary statements. Be sure to use multiple representations to justify each statement. Be prepared to share your summary statements with the class.

5-43. Some types of sequences have special names.

a. When the sequence generator *adds* a constant to each previous term, it is called an **arithmetic sequence**. Which of your sequences from problem 5-41 fall into this family? Should you include the sequence labeled (f) in this family? Why or why not? **[Sequences (a), (d), and (f) are arithmetic, and (f) is arithmetic because subtracting 2 is the same as adding –2.]**



b. When the sequence generator *multiplies* a constant times each previous term, it is called a **geometric sequence**. Which of the sequences from problem 5-41 are geometric? Should sequence (h) be in this group? Why or why not? **[Sequences (b), (g), (h), and (j) are geometric, and (g) and (h) are geometric because each term is the previous term multiplied by $\frac{1}{2}$ and $\frac{1}{3}$, respectively.]**

Review & Preview

5-44. Find the slope of the line you would get if you graphed each sequence listed below and connected the points.

a. $5, 8, 11, 14, \ldots$ **[$m = 3$]**

b. $3, 9, 15, \ldots$ **[$m = 6$]**

c. $26, 21, 16, \ldots$ **[$m = -5$]**

d. $7, 8.5, 10, \ldots$ **[$m = 1.5$]**

5-45. For the line passing through the points $(-2, 1)$ and $(2, -11)$,

a. Calculate the slope of the line. **[–3]**

b. Find an equation of the line. **[$y = -3x - 5$]**

5-46. Allie is making 8 dozen chocolate-chip muffins for the Food Fair at school. The recipe she is using makes 3 dozen muffins. If the original recipe calls for 16 ounces of chocolate chips, how many ounces of chocolate chips does she need for her new amount? (Allie buys her chocolate chips in bulk and can measure them to the nearest ounce.) **[43 ounces]**

5-47. The area of a square is 225 square centimeters.

a. Make a diagram and determine the length of each side. **[15 cm]**

b. Use the Pythagorean theorem to find the length of its diagonal. **[$15\sqrt{2} \approx 21.21$ cm.]**

5-48. Refer to sequences (c) and (i) in problem 5-41.

a. How are these two sequences similar? **[One possible answer is that their growth grows linearly.]**

b. The numbers in the sequence in part (e) from problem 5-41 are called **Fibonacci numbers**. They are named after an Italian mathematician who discovered the sequence while studying how fast rabbits could breed. What is different about this sequence than the other three you discovered? **[Answers vary. Sample: The sequence of differences between terms turns out to be almost the same as the sequence itself.]**

5-49. Chelsea dropped a bouncy ball off the roof while Nery recorded its rebound height. The table at right shows their data. Note that the 0 in the "Bounce" column represents the starting height.

Bounce	Rebound Height
0	800 cm
1	475 cm
2	290 cm
3	175 cm
4	100 cm
5	60 cm

a. To what family does the function belong? Explain how you know. **[Exponential, because the ratio of one rebound to the next is roughly constant ≈ 0.6]**

b. Show the data as a sequence. Is the sequence arithmetic, geometric, quadratic, or something else? Justify your answer. **[Roughly geometric, because it has a multiplier (though students may say it is neither because the multiplier is not exact).]**

5-50. For the function $f(x) = \sqrt{3x-2}$, find the value of each expression below.

a. $f(1)$ **[1]**

b. $f(9)$ **[5]**

c. $f(4)$ **[$\sqrt{10} \approx 3.16$]**

d. $f(0)$ **[undefined]**

e. What value of x makes $f(x) = 6$? **[$x = \frac{38}{3} = 12\frac{2}{3}$]**

5-51. Simplify each expression below.

a. $y + 0.03y$ **[$1.03y$]**

b. $z - 0.2z$ **[$0.8z$]**

c. $x + 0.002x$ **[$1.002x$]**

5-52. A tank contains 8000 liters of water. Each day, half of the water in the tank is removed. How much water will be in the tank at the end of:

a. The 4th day? **[500 liters]**

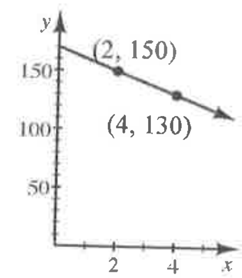
b. The 8th day? **[31.25 liters]**

5-53. Solve each system.

a. $y+3x=-10$ **[(−1, −7)]**
$5x-y=2$

b. $6x=7-2y$ **[($\frac{1}{2}$, 2)]**
$4x+y=4$

5-54. Draw a slope triangle and use it to find the equation of the line shown in the graph at right. **[$y=-10x+170$]**



5-55. This problem is a checkpoint for laws of exponents and scientific notation. It will be referred to as Checkpoint 5A.

Simplify each expression. In parts (e) through (f) write the final answer in scientific notation.

a. $4^2 \cdot 4^5$
[4^7]

b. $(5^0)^3$
[1]

c. $x^{-5} \cdot x^3$
[$x^{-2} = \frac{1}{x^2}$]

d. $(x^{-1} \cdot y^2)^3$
[$\frac{y^6}{x^3}$]

e. $(8 \times 10^5) \cdot (1.6 \times 10^{-2})$
[1.28×10^4]

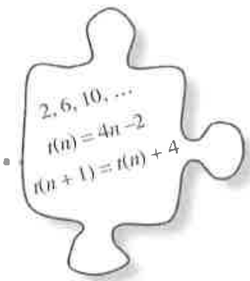
f. $\frac{4 \times 10^3}{5 \times 10^5}$
[8×10^{-3}]

Check your answers by referring to the Checkpoint 5A materials located at the back of your book.

If you needed help solving these problems correctly, then you need more practice. Review the Checkpoint 5A materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to do problems like these quickly and easily.

5.2.2 How do arithmetic sequences work?

Generalizing Arithmetic Sequences



In Lesson 5.2.1, you learned how to identify arithmetic and geometric sequences. Today you will solve problems involving arithmetic sequences. Use the questions below to help your team stay focused and start mathematical conversations.

What type of sequence is this? How do we know?

How can we find the equation?

Is there another way to see it?

5-56. LEARNING THE LANGUAGE OF SEQUENCES

Sequences have their own notation and vocabulary that help describe them, such as "term" and "term number." The questions below will help you learn more of this vocabulary and notation.

Consider the sequence $-9, -5, -1, 3, 7, \ldots$ as you complete parts (a) through (f) below.

a. Is this sequence arithmetic, geometric, or neither? How can you tell?
[It is arithmetic, since its generator is +4.]

b. What is the first term of the sequence? **[–9]**

c. When the sequence generator adds a number to each term, the value that is added is known as the **common difference**. It is the difference between each term and the term before it.

What is the sequence generator? **[Add 4.]**

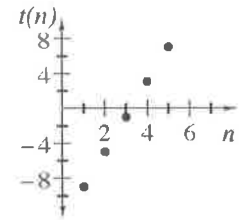
d. Record the sequence in a table. Remember a sequence table compares the term number, n, to the value of each term, $t(n)$. **[See table at right.]**

n	$t(n)$
1	–9
2	–5
3	–1
4	3
5	7

e. What is $t(n)$ when $n = 0$? **[–13]**

Problem continues on next page. →

5-56. *Problem continued from previous page.*



f. Graph the sequence. Should the graph be continuous or discrete? Why? **[It should be discrete, since the equation is for a sequence (see graph at right).]**

g. Write an equation (beginning $t(n) =$) for the n^{th} term of this sequence. **[$t(n) = 4n - 13$]**

h. What is the domain for the sequence equation that you have written? **[The domain is integers greater than or equal to one; the sequence starts at $n = 1$.]**

i. How is the **common difference** related to the graph and the equation? Why does this make sense? **[It is the slope of 4, since it represents the rate at which the sequence grows.]**

5-57. Consider the sequence $t(n) = -4, -1, 2, 5, \ldots$

a. If the first term is $t(1)$, what is $t(0)$ for this sequence? What is the common difference? **[$t(0) = -7$, 3]**

b. Write an equation for $t(n)$. Verify that your equation works for each of the first 4 terms of the sequence. **[$t(n) = 3n - 7$]**

c. Is it possible for $t(n)$ to equal 42? Justify your answer. **[No, because the domain of $t(n)$ includes only positive integers, none of which gives an output value of 42.]**

d. For the function $f(x) = 3x - 7$, is it possible for $f(x)$ to equal 42? Explain. **[Yes, $f(x)$ can be equal to 42 (when $x = \frac{46}{3}$), because it has a domain of all real numbers.]**

e. Explain the difference between $t(n)$ and $f(x)$ that makes your answers to parts (b) and (c) different. **[Part (b) has a domain limited to integers, while the domain of part (c) is all real numbers including non-integers.]**

5-58. Trixie wants to create an especially tricky arithmetic sequence. She wants the 5^{th} term of the sequence to equal 11 and the 50^{th} term to equal 371. That is, she wants $t(5) = 11$ and $t(50) = 371$. Is it possible to create an arithmetic sequence to fit her information? If it is possible, find the sequence generator, the initial value $t(0)$, and then find the equation for the arithmetic sequence. If it is not possible, explain why not. **[$t(n) = 8n - 29$, $t(0) = -29$, sequence generator = +8]**

5-59. Seven years ago, Kodi found a box of old baseball cards in the garage. Since then, he has added a consistent number of cards to the collection each year. He had 52 cards in the collection after 3 years and now has 108 cards.

a. How many cards were in the original box? Is this $t(0)$ or $t(1)$? Write the first few terms of the sequence. **[10 cards. $t(0)$. 24, 38, 52, 66, 80, ...]**

b. Kodi plans to keep the collection for a long time. How many cards will the collection contain 10 years from now? **[248 cards]**

c. Write an equation that determines the number of cards in the collection after n years. What does each number in your equation represent? **[$t(n) = 14n + 10$. 10 is the original number of cards (the starting value) and +14 is the sequence generator (or common difference).]**

5-60. Trixie now wants an arithmetic sequence with a sequence generator of –17 and a 16th term of 93. (In other words, $t(16) = 93$.) Is it possible to create an arithmetic sequence to fit her information? If it is possible, find the equation. If it is not possible, explain why not. **[$t(n) = -17n + 365$]**

5-61. Your favorite radio station, WCPM, is having a contest. The DJ poses a question to the listeners. If the caller answers correctly, he or she wins the prize money. If the caller answers incorrectly, \$20 is added to the prize money and the next caller is eligible to win. The current question is difficult, and no one has won for two days.

a. Lucky you! Fourteen people already called in today with incorrect answers, so when you called (with the right answer, of course) you won \$735! How much was the prize worth at the beginning of the day today? **[\$455]**

b. Suppose the contest always starts with \$100. How many people would have to guess incorrectly for the winner to get \$1360? **[63 people would have to guess incorrectly.]**

5-62. Trixie is at it again. This time she wants an arithmetic sequence that has a graph with a slope of 22. She also wants $t(8)=164$ and the 13[th] term to have a value of 300. Is it possible to create an arithmetic sequence to fit her information? If it is possible, find the equation. If it is not possible, explain why not. **[It is not possible. The graph of the equation for the two given terms would have a slope of 27.2, not 22.]**

5-63. Find the equation for each arithmetic sequence represented by the tables below. **[a: $t(n)=11n-23$, b: $t(n)=-3n+310$]**

a.

n	$t(n)$
7	54
3	10
19	186
16	153
40	417

b.

n	$t(n)$
100	10
70	100

5-64. Trixie exclaimed, *"Hey! Arithmetic sequences are just another name for linear functions."* What do you think? Justify your idea based on multiple representations. **[Answers vary, but students may notice that terms in the sequence may be treated as points, e.g., $t(3) = 12$ and $t(8) = 22$ become (3, 12) and (8, 22).]**

Review & Preview

5-65. Determine whether 447 is a term of each sequence below. If so, which term is it?

a. $t(n)=5n-3$
[Yes, the 90[th] term or $t(90) = 447$]

b. $t(n)=24-5n$
[No]

c. $t(n)=-6+3(n-1)$
[Yes, the 152[nd] term or $t(152) = 447$]

d. $t(n)=14-3n$
[No]

e. $t(n)=-8-7(n-1)$ **[No, $n = -64$ is not in the domain.]**

5-66. Choose one of the sequences in problem 5-65 for which you determined that 447 is *not* a term. Write a clear explanation describing how you can be sure that 447 is not a term of the sequence. **[Justifications vary.]**

5-67. Find the sequence generator for each sequence listed below. Write an equation for the n^{th} term in each sequence below, keeping in mind that the first term of each sequence is $t(1)$.

a. $4, 7, 10, 13, \ldots$
[$m = 3$, $t(n) = 3n + 1$]

b. $3, 8, 13, \ldots$
[$m = 5$, $t(n) = 5n - 2$]

c. $24, 19, 14, \ldots$
[$m = -5$, $t(n) = -5n + 29$]

d. $7, 9.5, 12, \ldots$
[$m = 2.5$, $t(n) = 2.5n + 4.5$]

5-68. Great Amusements Park has been raising its ticket prices every year, as shown in the table at right.

Year	Price
0	$50
1	$55
2	$60.50
3	$66.55

a. Describe how the ticket prices are growing. **[Descriptions vary, but students may say they are multiplying by 1.1 or growing by 10% each year.]**

b. What will the price of admission be in year 6? **[$88.58]**

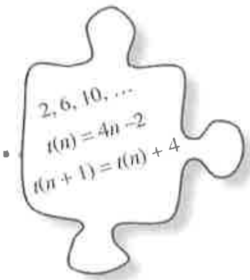
5-69. Solve the system at right for m and b. **[$m = 13, b = 17$]**

$$1239 = 94m + b$$
$$810 = 61m + b$$

5-70. Write an equation or system of equations and solve the problem below.
[$5b + 3h = 339$, $b = h + 15$; 48 bouquets and 33 hearts]

The French club sold rose bouquets and chocolate hearts for Valentine's Day. The roses sold for $5 and the hearts sold for $3. The number of bouquets sold was 15 more than the number of hearts sold. If the club collected a total of $339, how many of each gift was sold.

5.2.3 How else can I write the equation?



Recursive Sequences

In this chapter you have been writing equations for arithmetic sequences so that you could find the value of any term in the sequence, such as the 100^{th} term, directly. Today you will investigate recursive sequences. A term in a recursive sequence depends on the term(s) before it.

5-71. Look at the following sequence:

$$-8, -2, 4, 10, \ldots$$

a. What are two ways that you could find the 10^{th} term of the sequence? What is the 10^{th} term? **[Possible responses: Use the equation $t(n) = 6n - 14$ and evaluate for n=10, or keep adding 6 until you get to the 10^{th} term. 74.]**

b. If you have not done so already, write an equation that lets you find the value of any term $t(n)$. This kind of equation is called an **explicit equation.** **[$t(n) = 6n - 14$]**

c. The next term after $t(n)$ is called $t(n+1)$. Write an equation to find $t(n+1)$ if you know what $t(n)$ is. An equation that depends on knowing other terms is called a **recursive equation**. **[$t(n+1) = t(n) + 6$]**

5-72. Alejandro used the recursive equation from part (c) of problem 5-71 to write a sequence and came up with the following sequence:

$$0, 6, 12, 18, 24$$

a. Does Alejandro's sequence match the recursive equation from problem 5-71? **[No.]**

b. Why did Alejandro get a different sequence than the one from problem 5-71? How can you mathematically write down the information he needs so that he can write the correct sequence? **[He started with the first term of 0; to create the same sequence he needs to use a first term of –8. $t(1) = -8$ or $t(0) = -14$.]**

5-73. Avery and Collin were trying to challenge each other with equations for sequences. Avery wrote:

$$t(n+1)=t(n)^2-1$$
$$t(1)=3$$

a. Help Collin write the first 4 terms of this sequence. **[3, 8, 63, 3968]**

b. Is Avery's sequence arithmetic, geometric, or some other kind of sequence? How do you know? **[This sequence is not arithmetic nor geometric (nor quadratic).]**

c. Describe to Collin how he could find the 10^{th} term of this sequence. You do not need to actually find the 10^{th} term. **[He would need to find all of the terms up to the 9^{th} term to find the 10^{th} term.]**

5-74. Avery and Collin were still at it.

a. Collin wrote: $t(2)=19$

$$t(n+1)=t(n)-2$$

Help Avery write an explicit equation. Is the sequence arithmetic, geometric, or neither? **[$t(n)=2n-15$. The sequence is arithmetic.]**

b. Then Avery wrote $t(n)=6n+8$. Help Collin write a recursive equation. **[$t(1)=14$; $t(n+1)=t(n)+6$]**

5-75. The Fibonacci sequence is a famous sequence that appears many times in mathematics. It can describe patterns found in nature, such as the number of petals on flowers, the arrangements of seeds in sunflowers, or scales on pinecones. It is named after Leonardo of Pisa, who was known as Fibonacci. He introduced the sequence to Western European mathematicians in 1202, though it had been described earlier by others including mathematicians in India.

The equation that describes the Fibonacci sequence can be written as:

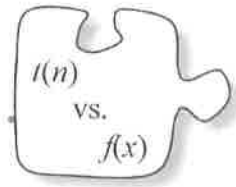
$$t(1)=1$$
$$t(2)=1$$
$$t(n+1)=t(n)+t(n-1)$$

a. Write the first 10 terms of the Fibonacci sequence. **[1, 1, 2, 3, 5, 8, 13, 21, 34, 55]**

b. Is the Fibonacci sequence arithmetic, geometric, or neither? **[Neither.]**

c. Describe what you would need to do in order to find the 100^{th} term of the Fibonacci sequence. Do not actually calculate the 100^{th} term. **[Find all the terms up to the 98^{th} and 99^{th} terms, and then add the 98^{th} and 99^{th} terms.]**

5.3.1 What is the rate of change?



Patterns of Growth in Tables and Graphs

So far in this chapter you have looked at several types of sequences and compared linear and exponential growth patterns in situations, tables, and graphs. In this lesson you will compare patterns of growth rates to each other. This work will also help you write equations for exponential sequences in the next lesson.

5-82. PATTERNS OF GROWTH

Sequence A	
n	$t(n)$
1	27
2	54
3	81
4	108

Sequence B	
n	$t(n)$
1	9
2	36
3	81
4	144

Sequence C	
n	$t(n)$
1	6
2	12
3	24
4	48

Your Task:

- Represent these three sequences on a graph (the Lesson 5.3.1A Resource Page). Use a different color for each sequence. Although the graph is discrete, connect the lines so you can see the trends easier.
- Consider the discussion points below for each sequence as you investigate the growth of these three sequences. You can discuss the sequences in any order.
- Be prepared to share your results with the class.

Discussion Points

How do the inputs, n, and the outputs of the sequence generator, $t(n)$, increase?

How does the sequence grow? Is the rate of change constant or changing? How? (You can make growth triangles to help answer this question.)

If you knew a specific term, how would you find the next term? For example, if you knew the 10^{th} term, could you find the 11^{th} term?

Which family of functions best models each sequence?

[See the "Suggested Lesson Activity" for possible student responses.]

5-83. GROWTH RATES IN SEQUENCES

Consider how fast each of the sequences is growing by looking at the tables and the graph. Do not make any additional computations. Instead make conjectures based on the tables and graphs.

a. If n represents the number of years, and $t(n)$ represents the amount of money in your savings account, which account would you want, Sequence A, B, or C? **[After one or two years, Sequence A had the most growth. After four years, Sequence B had the most growth.]**

b. Would your answer change if you kept the account for many years? **[Most students will choose Sequence B because it appears to be growing the fastest; Sequence B has the steepest line after four years.]**

c. Obtain the Lesson 5.3.1B Resource Page from your teacher. Extend the tables and the graph to $n = 7$. The table for Sequence B has been completed for you. **[Solution graphs are provided with the resource pages.]**

d. Based on your new graph, do you want to change your answer to part (b)? Why or why not? **[Students should notice that Sequence C is growing the fastest, and will soon overtake Sequence B. Sequence C will have shown the most growth after 8 years.]**

5-84. WHICH GROWS THE MOST?

a. Will an exponentially growing bank account eventually contain more money than a linearly growing bank account for the same amount of initial savings, no matter how fast (steep) the rate of growth of the linear account? Use the slope triangles on your graph from problem 5-83 to help you explain. **[Yes. See the "Suggested Lesson Activity" for possible explanations.]**

b. How does the growth of a quadratic sequence like Sequence B compare to exponential growth? **[See the "Suggested Lesson Activity" for possible explanations.]**

Review & Preview

5-85. Identify the following sequences as linear, exponential, or other. For the linear and exponential sequences, identify the rate of change and whether it is a constant that is added or multiplied.

a. $12, 144, 1728, \ldots$ **[exponential, multiply by 12]**

b. $0, 5, 10, 15, 20, 25, \ldots$ **[linear, add 5]**

c. $0, 4, 16, 36, 64, \ldots$ **[other (quadratic)]**

d. $1.5, 2.25, 3.375, 5.0625, \ldots$ **[exponential, multiply by 1.5]**

5-86. Solve the system of equations at right. **[(2, –4)]**

$$y = -x - 2$$
$$5x - 3y = 22$$

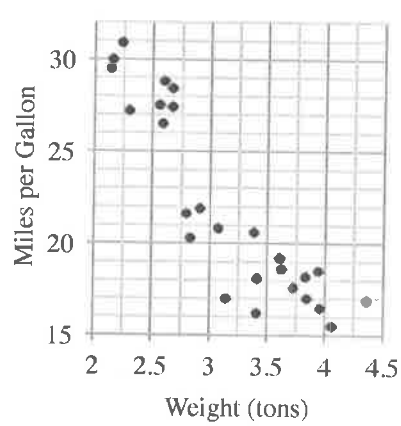
5-87. Write the first five terms of each sequence.

a. $t(1) = -3$, $t(n+1) = -2 \cdot t(n)$ **[–3, 6, –12, 24, –48]**

b. $t(1) = 8$, $t(n+1) = t(n) - 5$ **[8, 3, –2, –7, –12]**

c. $t(1) = 2$, $t(n+1) = (t(n))^{-1}$ **[$2, \frac{1}{2}, 2, \frac{1}{2}, 2$]**

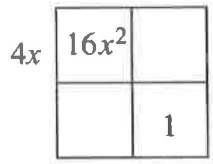
5-88. The graph at right compares the gas mileage to the weight of numerous vehicles.



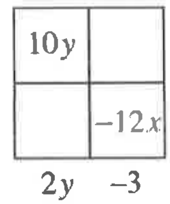
Describe the association between these two quantities. **[Moderate negative linear association with no outliers. The data appear to be in two clusters, probably indicating two classes of vehicles.]**

5-89. Find the missing areas and dimensions for each generic rectangle below. Then write each area as a sum and as a product.

a.



b.



[$(4x+1)(4x+1)=16x^2+8x+1$] **[$(4x+5)(2y-3)=8xy-12x+10y-15$]**

5-90. This problem is a checkpoint for writing the equation of a line. It will be referred to as Checkpoint 5B.

Use the given information to find an equation of the line. **[a: $y=2x-3$; b: $y=-3x-1$; c: $y=\frac{2}{3}x-2$; d: $y\approx\frac{5}{2}x+9$]**

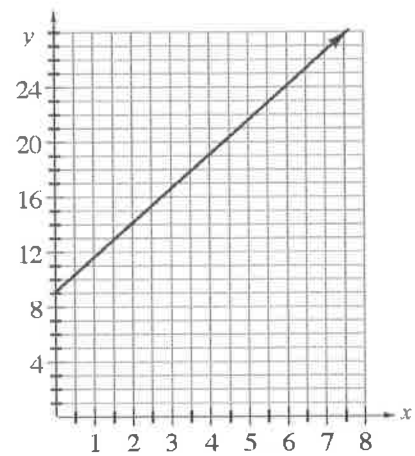
a. Slope 2 and passing through $(10,17)$.

b. Passing through $(1,-4)$ and $(-2,5)$.

c.

x	−6	−3	0	3	6
y	−6	−4	−2	0	2

d.



Check your answers by referring to the Checkpoint 5B materials located at the back of your book.

If you needed help solving these problems correctly, then you need more practice. Review the Checkpoint 5B materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to do problems like these quickly and easily.

5.3.2 How can I use a multiplier?

Using Multipliers to Solve Problems

In the past few lessons, you have investigated sequences that grow by adding (arithmetic) and sequences that grow by multiplying (geometric). In today's lesson, you will learn more about growth by multiplication as you use your understanding of geometric sequences and multipliers to solve problems. As you work, use the following questions to move your team's discussion forward:

What type of sequence is this? How do we know?

How can we describe the growth?

How can we be sure that our multiplier is correct?

5-91. Thanks to the millions of teens around the world seeking to be just like their math teachers, industry analysts predict that sales of the new πPhone will skyrocket!

πPHONES SWEEP THE NATION

Millions demand one!

(API) – Teenagers and Hollywood celebrities flocked to an exclusive shop in Beverly Hills, California yesterday, clamoring for the new πPhone. The store expects to start by selling 100 and expects to sell an average of 15% more each week after that.

"I plan to stand in line all night!" said Nelly Hillman. "As soon as I own one, I'll be cooler than everyone else."

Across the globe, millions of fans

a. The article provides a model for how many πPhones the store expects to sell. They start by selling 100 πPhone pre-orders in week zero. Predict the number sold in the 4^{th} week. **[About 175.]**

b. If you were to write the number of πPhones the store received each week as a sequence, would your sequence be arithmetic, geometric, or something else? Justify your answer. **[The sequence would be geometric; the sequence grows by multiplying by 1.15.]**

c. The store needs to know how many phones to order for the last week of the year. If you knew the number of πPhones sold in week 51 how could you find the sales for week 52? Write a recursive equation to show the predicted sales of πPhones in the n^{th} week. **[Multiply week 51 by 1.15. $t(1) = 100$, $t(n+1) = t(n) \cdot 1.15$, or $t(n+1) = t(n) + 0.15t(n)$]**

Problem continues on next page. →

5-91. *Problem continued from previous page.*

d. Write an explicit equation that starts with "$t(n)=$" to find the number of πPhones sold during the n^{th} week without finding all of the weeks in between. **[$t(n)=100\cdot1.15^n$]**

e. How many πPhones will the store predict it sells in the 52^{nd} week? **[$100(1.15)^{52}\approx 143\,313$ πPhones]**

5-92. A new πRoid, a rival to the πPhone, is about to be introduced. It is cheaper than the πPhone, so more are expected to sell. The manufacturer plans to make and then sell 10,000 pre-orders in week zero and expects sales to increase by 7% each week.

a. Write an explicit and a recursive equation for the number of πRoids sold during the n^{th} week. **[$t(n)=10\,000(1.07)^n$, $t(n+1)=t(n)\cdot1.07$ and $t(0)=10{,}000$.]**

b. What if the expected weekly sales increase were 17% instead of 7%? Now what would the new explicit equation be? How would it change the recursive equation? **[$t(n)=10\,000(1.17)^n$. 1.07 would become 1.17.]**

5-93. Oh no! Thanks to the lower price, 10,000 πRoid were made and sold initially, but after that, weekly sales actually decreased by 3%.

a. Find an explicit and a recursive equation that models the product's actual weekly sales. **[$t(n)=10\,000(0.97)^n$; $t(0)=10{,}000$ and $t(n+1)=t(n)\cdot0.97$]**

5-94. In a geometric sequence, the sequence generator is the number that one term is multiplied by to generate the next term. Another name for this number is the **multiplier**.

a. Look back at your work for problems 5-92, and 5-93. What is the multiplier in each of these three situations? **[1.07, 1.17, and 0.97.]**

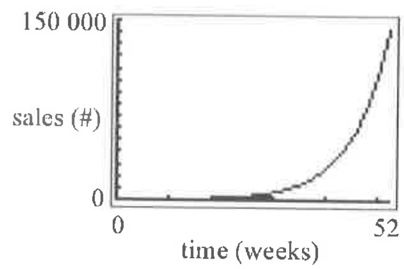
b. What is the multiplier for the sequence 8, 8, 8, 8, ... ? **[1]**

c. Explain what happens to the terms of the sequence when the multiplier is less than 1, but greater than zero. What happens when the multiplier is greater than 1? Add this description to your Learning Log. Title this entry "Multipliers" and add today's date. **[Students should notice that a sequence grows if its multiplier is greater than 1 and shrinks if its multiplier is between 0 and 1.]**

5-95. MULTIPLE REPRESENTATIONS ON THE GRAPHING CALCULATOR

a. According to the model in problem 5-93, how many weeks will it take for the weekly sales to drop to only one πRoid per week? Make a conjecture.

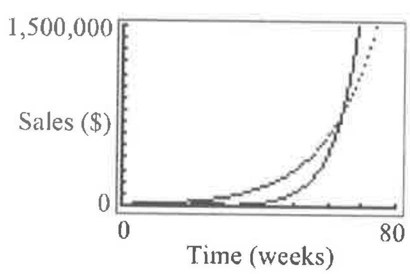
b. Before calculating the exact answer to the question in part (a), become comfortable with using your graphing calculator. On your calculator, make a graph for the sales of πPhones (problem 5-91) for the first year. Sketch the graph on your paper. Make sure you show the scale of the axes on your sketch. **[Graph follows. See the "Suggested Lesson Activity" for additional calculator information.]**



c. Use the table on your calculator to determine where, if at all, the graph in part (b) crosses the x-axis. **[The curve does not cross the x-axis.]**

d. Enter the explicit equations for both problems 5-91, $t(n) = 100 \cdot 1.15^n$, and problem 5-92, $t(n) = 10\,000 \cdot 1.07^x$, in your calculator. Use your table to find the number of weeks it takes for sales in the first equation to exceed the sales in the second equation. **[64 weeks.]**

e. Make a sketch of the graph of both equations in part (d). Be sure to show the point of intersection. Label the scale on both axes. **[Graph follows. See the "Suggested Lesson Activity" for additional calculator information.]**



f. Now use your calculator to answer the question in part (a). How close was your conjecture? **[After 302 weeks.]**

5-96. Write an explicit and a recursive equation for each table below. Be sure to check that your equations work for all of the entries in the table. **[a: $t(n) = 1600(1.25)^n$, $a_1 = 1600$ and $a_{n+1} = a_n \cdot 1.25$; b: $t(n) = 3906.25(0.8)^n$, $a_1 = 3906.25$ and $a_{n+1} = a_n \cdot 0.8$; c: $t(n) = 50(1.44)^n$, $a_1 = 50$ and $a_{n+1} = a_n \cdot 1.44$; d: $t(n) \approx 41.67(1.2)^n$, $a_1 = 1600$ and $a_{n+1} = a_n \cdot 1.25$.]**

a.

n	$t(n)$
0	1600
1	2000
2	2500
3	3125
4	3906.25

b.

n	$t(n)$
0	3906.25
1	3125
2	2500
3	2000
4	1600

c.

n	$t(n)$
0	50
1	72
2	103.68
3	149.2992

d.

n	$t(n)$
0	
1	50
2	
3	72
4	
5	103.68
6	
7	149.2992

e. How are the tables in (a) and (b) related? How are the multipliers for (a) and (b) related? Why does this make sense? **[The tables are the same but with the order of the outputs reversed. The multipliers in part (a) and (b) are reciprocals.]**

f. What strategies did you use to find the equation for part (d)? How is the table in part (d) related to the one in part (c)? **[Multiplying by the same unknown twice, or multiplying by $\sqrt{1.44}$.]**

g. In part (d), why is term 2 *not* 61? **[Because the growth is not linear (or arithmetic).]**

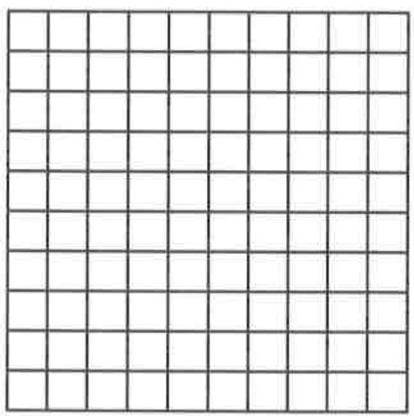
5-97. PERCENTS AS MULTIPLIERS

What a deal! Just deShirts is having a 20% off sale. Trixie rushes to the store and buys 14 shirts. When the clerk rings up her purchases, Trixie sees that the clerk has added the 5% sales tax first, before taking the discount. Trixie wonders whether adding the sales tax before the discount makes her final cost more than adding the sales tax after the discount. Without making any calculations, make a conjecture. Is Trixie getting charged more when the clerk adds sales tax first? The next few problems will help you figure it out for sure. **[Conjectures vary.]**

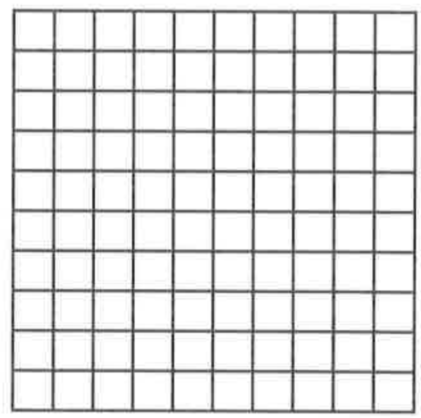
5-98. Karen works for a department store and receives a 20% discount on any purchases that she makes. The department store is having a clearance sale, and every item will be marked 30% off the regular price. Karen has decided to buy the $100 dress she's been wanting. When she includes her employee discount with the sale discount, what is the total discount she will receive? Does it matter what discount she takes first? Use the questions below to help you answer this question.

a. Use the grids like the ones below to picture another way to think about this situation. Using graph paper, create two 10-by-10 grids (as shown below) to represent the $100 price of the dress.

CASE 1: 20% discount first



CASE 2: 30% discount first



b. Use the first grid to represent the 20% discount followed by the 30% discount (Case 1). Use one color to shade the number of squares that represent the first 20% discount. For whatever is left (unshaded), find the 30% discount and use another color to shade the corresponding number of squares to represent this second discount. Then repeat the process (using the other grid) for the discounts in reverse order (Case 2).

c. How many squares remain after the first discount in Case 1? In Case 2?
[Case 1: 80 squares; Case 2: 70 squares]

d. How many squares remain after the second discount in Case 1? In Case 2?
[Case 1: 56 squares; Case 2: 56 squares]

e. Explain why these results make sense. **[The results should be the same because multiplication is commutative.** $\mathbf{0.80(0.70) = 0.70(0.80) = 0.56}$ **.]**

5-99. Suppose that Trixie's shirts cost x dollars in problem 5-97.

a. If x represents the cost, how could you represent the tax? How could you represent the cost plus the tax? **[$0.05x$. $x + 0.05x$ or $1.05x$]**

b. How could you represent the discount? How could you represent the cost of the shirt after the discount? **[$0.20x$, $x - 0.20x = 0.80x$]**

c. Did Trixie get charged more because the clerk added the sales tax first? Justify your reasoning. **[No, because the order does not matter. $0.80(1.05x) = 1.05(0.80x)$. Most students are surprised that in both situations, Trixie is charged the same.]**

5-100. Remember the "Multiplying Like Bunnies" problem at the beginning of this chapter? In that problem, Lenny and George started with 2 rabbits and each month the number of rabbits that they had doubled since each pair of rabbits produced another pair of rabbits.

a. Find an equation for this situation. Let y represent the number of rabbits after x months. **[$y = 2 \cdot 2^x$]**

b. Lenny and George now have over 30 million rabbits. How many months have passed? **[24 months]**

c. With 30 million rabbits, the bunny farm is getting overcrowded and some of the rabbits are dying from a contagious disease. The rabbits have stopped reproducing, and the disease is reducing the total rabbit population at a rate of about 30% each month. If this continues, then in how many months will the population drop below 100 rabbits? **[36 months]**

Methods and Meanings

Math Notes

Types of Sequences

An **arithmetic sequence** is a sequence with an addition (or subtraction) **sequence generator**. The number added to each term to get the next term is called the **common difference**..

A **geometric sequence** is a sequence with a multiplication (or division) generator. The number multiplied by each term to get the next term is called the **common ratio** or the **multiplier**.

A multiplier can also be used to increase or decrease by a given percentage. For example, the multiplier for an increase of 7% is 1.07. The multiplier for a decrease of 7% is 0.93.

A **recursive sequence** is a sequence in which each term depends on the term(s) before it. The equation of a recursive sequence requires at least one term to be specified. A recursive sequence can be arithmetic, geometric, or neither.

For example, the sequence $-1, 2, 5, 26, 677, \ldots$ can be defined by the **recursive equation**:

$$t(1) = -1, \quad t(n+1) = \left(t(n)\right)^2 + 1$$

An alternative notation for the equation of the sequence above is:

$$a_1 = -1, \quad a_{n+1} = (a_n)^2 + 1$$

Review & Preview

5-101. For each table below, find the missing entries and write an equation.

a. [$y = 2 \cdot 4^x$]

Month (x)	0	1	2	3	4	5	6
Population (y)	2	8	32	**128**	**512**	**2048**	**8192**

b. [$y = 5 \cdot (1.2)^x$]

Year (x)	0	1	2	3	4	5	6
Population (y)	5	6	7.2	**~8.6**	**~10.4**	**~12.4**	**~14.9**

5-102. Convert each percent increase or decrease into a multiplier.

a. 3% increase **[1.03]**

b. 25% decrease **[0.75]**

c. 13% decrease **[0.87]**

d. 2.08% increase **[1.0208]**

5-103. Mr. C is such a mean teacher! The next time Mathias gets in trouble, Mr. C has designed a special detention for him. Mathias will have to go out into the hall and stand exactly 100 meters away from the exit door and pause for a minute. Then he is allowed to walk exactly halfway to the door and pause for another minute. Then he can again walk exactly half the remaining distance to the door and pause again, and so on. Mr. C says that when Mathias reaches the door he can leave, *unless* he breaks the rules and goes more than halfway, even by a tiny amount. When can Mathias leave? Prove your answer using multiple representations. **[Technically, Mathias can never leave, either because he will never reach the door or because he cannot avoid breaking the rules. The equation for this situation is $y = 100(0.5)^x$, where x is the number of minutes that have passed and y is the distance (in meters) from the door.]**

5-104. Simplify each expression.

a. $(2m^3)(4m^2)$ **[$8m^5$]**

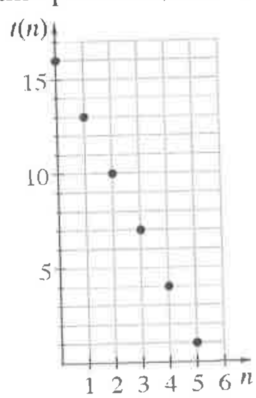
b. $\frac{6y^5}{3y^2}$ **[$2y^3$]**

c. $\frac{-4y^2}{6y^7}$ **[$\frac{-2}{3y^5}$]**

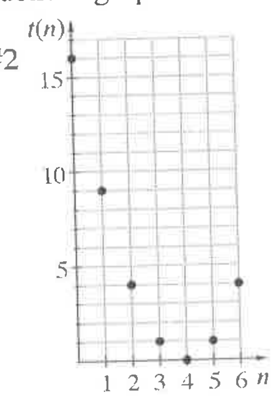
d. $(-2x^2)^3$ **[$-8x^6$]**

5-105. For this problem, refer to the sequences graphed below.

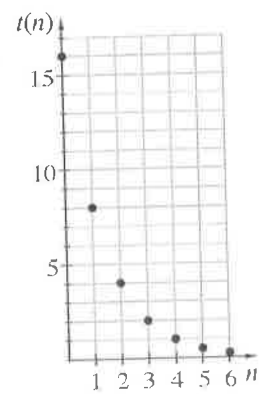
\#1



\#2



\#3



a. Identify each sequence as arithmetic, geometric, or neither.
[#1 is arithmetic, #2 is neither, #3 is geometric]

b. If it is arithmetic or geometric, describe the sequence generator.
[#1 the generator is to add –3, #3 the generator is to multiply by $\frac{1}{2}$]