4.1.1 How can I use variables to solve problems?

$$6x + 2y = 4$$
$$y = 2x - 7$$

Solving Word Problems by Writing Equations
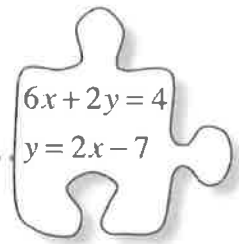
Today you will learn to translate written information into algebraic symbols and then solve the equations that represent the relationships.

4-1.   Match each **mathematical sentence** on the left with its translation on the right.

a. $2z + 12 = 30$

b. $12z + 5(z + 2) = 30$

c. $z + (z - 2) + 5(z - 2) = 30$

d. $z + 12z = 30$

1.   A zoo has two fewer elephants than zebras and five times more monkeys than elephants. The total number of elephants, monkeys, and zebras is 30. **[C]**

2.   Zola earned $30 by working two hours and receiving a $12 bonus. **[A]**

3.   Thirty ounces of metal is created by mixing zinc with silver. The number of ounces of silver needed is twelve times the number of ounces of zinc. **[D]**

4.   Eddie, who earns $5 per hour, worked two hours longer than Zach, who earns $12 per hour. Together they earned $30. **[B]**

4-2.   Mathematical sentences, like those in the left column of problem 4-1, are easier to understand when everyone knows what the variables represent. A statement that describes what the variable represents is called a **"let" statement**. For example, for mathematical sentence (a) above, which is matched with translation 2, we could say, "Let z = the amount of time Zola worked (in hours)." Note that a "let" statement always indicates the units of measurement.

Write a "let" statement for each of the mathematical sentences in parts (b) through (d) above. **[b: Let z = amount of time Zach worked (hours), c: Let z = the number of zebras (counts do not have units), d: Let z = weight of zinc (ounces)]**

4-3.   The perimeter of a triangle is 31 cm. Sides #1 and #2 have equal length, while Side #3 is one centimeter shorter than twice the length of Side #1. Let's determine how long each side is:

a. Let x represent the length of Side #1. What essential part of this "let" statement is missing? What is the length of Side #2? Side #3? [**The units of measurement, centimeters. Side #2 = x, Side #3 = $2x - 1$**]

b. Write a mathematical sentence that states that the perimeter is 31 cm. [$x + x + (2x - 1) = 31$]

c. Solve the equation you found in part (b) and determine the length of each side. Be sure to label your answers with the appropriate units. [$x = 8$, **so Side #1 = Side #2 = 8 cm and Side #3 = $2 \cdot 8 - 1 = 15$ cm.**]

4-4.   THE ENVRONMENTAL CLUB

Charles and Amy are part of the Environmental Club at school. Their service project for the semester is to get a tree for their school donated and plant it on the school grounds. Charles' uncle owns a tree nursery and is willing to donate a 3-foot tall tree that he says will grow 1.5 feet per year.

Amy goes to another nursery in town, but is only able to get tree seeds donated. According to the seed package, the tree will grow 1.75 feet per year. Charles plants his tree and Amy plants a seed on the same day. Amy thinks that even though her tree will be much shorter than Charles' tree for the first several years, it will eventually be taller because it grows more each year, but she does not know how many years it will take for her tree to get as tall as Charles' tree.

Will the trees ever be the same size? If so, how many years will it take?

Your Task:

- Represent this problem with tables, equations, and one graph.

- Use *each* representation to find the solution. Explain how you found the solution in each of the three representations.

Discussion Points

How can the given information be represented with equations?

What is a solution to a two-variable equation?

How can this problem be represented on one graph?

How does the solution appear on the graph?

[$y = 1.5x + 3$, $y = 1.75x$; **The trees will be 12 years old and 21 feet tall.**]

Core Connections Algebra

4-5.   In the previous problem, you wrote equations that were **models** of a real-life situation. **Models** are usually not perfect representations, but they are useful for describing real-life behavior and for making predictions. You predicted when the two trees would be the same height.

   a.   What are an appropriate domain and range for the two models of tree growth? **[See the "Suggested Lesson Activity" for discussion.]**

   b.   Where can you find the y-intercepts of the model on the graph, in the table, and in the equation? In the situation of the story, what does the y-intercept represent? **[At the y-axis on the graph, where $x = 0$ in the table, and the constant in the equation. The y-intercepts represent the height of the tree in feet when it was planted at the school.]**

4-6.   For the following word problems, write one or two equations. Be sure to define your variable(s) and units of measurement with appropriate "let" statements and label your answers. You do not need to solve the equations yet.

   a.   After the math contest, Basil noticed that there were four extra-large pizzas that were left untouched. In addition, another three slices of pizza were uneaten. If there were a total of 51 slices of pizza left, how many slices does an extra-large pizza have? **[Let s = number of slices on an extra-large pizza. $4s + 3 = 51$]**
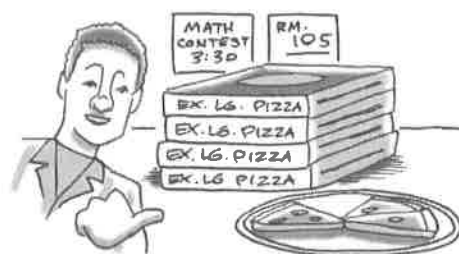
   b.   Herman and Jacquita are each saving money to pay for college. Herman currently has $15,000 and is working hard to save $1000 per month. Jacquita only has $12,000 but is saving $1300 per month. In how many months will they have the same amount of savings? **[Let m = time they have been saving (months), h = amount Herman has saved ($), and j = amount Jacquita has saved ($). Students could write one equation: $15,000 + 1000m = 12,000 + 1300m$, or they could write two equations: $h = 15,000 + 1000m$ and $j = 12,000 + 1300m$ set them equal to represent when $h = j$.]**

   c.   George bought some CDs at his local store. He paid $15.95 for each CD. Nora bought the same number of CDs from a store online. She paid $13.95 for each CD, but had to pay $8 for shipping. In the end, both George and Nora spent the exact same amount of money buying their CDs! How many CDs did George buy? **[Let c = number of CDs that George buys. $15.95c = 13.95c + 8$, OR, let g = money George spent ($) and n = money Nora spent ($) then $g = 15.95c$ and $n = 13.95c + 8$ and set them equal to represent when $g = n$.]**

4-7. Solve part (b) of problem 4-6 above. In how many months would they have the same amount of savings? How much savings would they have at that time? **[10 months, $25,000]**


METHODS AND MEANINGS

Line of Best Fit

A **line of best fit** is a straight line that represents, in general, data on a scatterplot, as shown in the diagram. This line does not need to touch any of the actual data points, nor does it need to go through the origin. The line of best fit is a model of numerical two-variable data that helps describe the data in order to make predictions for other data.
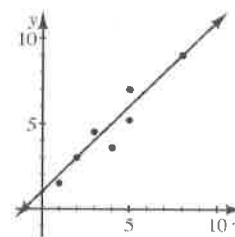

To write the equation of a line of best fit, find the coordinates of any two convenient points on the line (they could be lattice points where the gridlines intersect, or they could be data points, or the origin, or a combination). Then write the equation of the line that goes through these two points.

Review & Preview

4-8. Smallville High School Principal is concerned about his school's Advanced Placement (AP) test scores. He wonders if there is a relationship between the students' performance in class and their AP test scores so he randomly selects a sample of ten students who took AP examinations and compares their final exam scores to their AP test scores.

AP Score	5	1	4	2	1	4	2	1	3	5
Final%	97	70	84	66	62	79	73	63	82	90

Create a scatterplot on graph paper. Draw a line of best fit that represents the data. Refer to the Math Notes box in this lesson. Use the equation of your line of best fit to predict the final exam score of another Smallville HS student who scored a 3 on their AP test. **[Approximately $f = 58 + 7a$ where f is the final exam score (in percent) and a is the AP score; about 79%]**
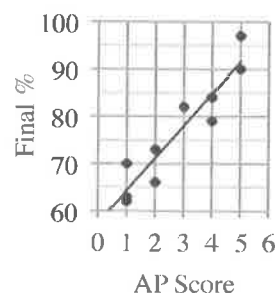
4-9.   Solve for x. Check your solutions, if possible.

a. $-2(4-3x)-6x=10$
      [no solution]

b. $\frac{x-5}{-2} = \frac{x-1}{-3}$
      [$x = 13$]

4-10.   On the same set of axes, use slope and y-intercept to graph each
        line in the system shown at right. Then find the point(s) of
        intersection, if one (or more) exists. **[(−1, 3)]**

        $y = -x + 2$
        $y = 3x + 6$

4-11.   A team of students is trying to answer the scientific notation
        problem $2 \times 10^3 \cdot 4 \times 10^7$.

        Jorge thinks they should use a generic rectangle because there are two
        terms multiplied by two terms.

        Cadel thinks the answer is 8×10^{10} but he cannot explain why.

        Lauren thinks they should multiply the like parts. Her answer is 8×100^{21}.

        Who is correct? Explain why each student is correct or incorrect. **[Cadel is correct
        because he followed the exponent rules. Jorge is incorrect; the problem only
        contains multiplication, so there are not two terms and the Distributive Property
        cannot be used. Lauren did not follow the exponent rules.]**

4-12.   For each of the following generic rectangles, find the dimensions (length and width)
        and write the area as the product of the dimensions and as a sum.

a.   **[$3y(y-4)=3y^2-12y$]**

$3y^2$	$-12y$

b.   **[$(3y+5)(y-4)=3y^2-7y-20$]**

$3y^2$	$-12y$
$5y$	-20

4-13.   A **prime number** is defined as a number with exactly two integer factors: itself and
        1. Jeannie thinks that all prime numbers are odd. Is she correct? If so, state how you
        know. If not, provide a counterexample. **[No; 2 is a prime number and it is even.]**

4-14. Solve this problem by writing and solving an equation. Be sure to define your variable. [If $x =$ the length, $2(x) + 2(3x - 1) = 30$, width is 4 in., length is 11 in.]

A rectangle has a perimeter of 30 inches. Its length is one less than three times its width. What are the length and width of the rectangle?

4-15. The basketball coach at Washington High School normally starts each game with the following five players:

Melinda, Samantha, Carly, Allison, and Kendra

However, due to illness, she needs to substitute Barbara for Allison and Lakeisha for Melinda at this week's game. What will be the starting roster for this upcoming game? [Lakeisha, Samantha, Carly, Barbara, and Kendra]

4-16. When Ms. Shreve solved an equation in class, she checked her solution and it did not make the equation true! Examine her work below and find her mistake. Then find the correct solution. [She combined terms from opposite sides of the equation. Instead, line 4 should read $2x = 14$, so $x = 7$ is the solution.]

$$5(2x - 1) - 3x = 5x + 9$$
$$10x - 5 - 3x = 5x + 9$$
$$7x - 5 = 5x + 9$$
$$12x = 4$$
$$x = \frac{1}{3}$$

4-17. Determine if the statement below is always, sometimes, or never true. Justify your conclusion. [This statement is sometimes true. It is true when $x = 0$, but otherwise it is false because the Distributive Property states that $a(b + c) = ab + ac$. Students can also justify this with a diagram of algebra tiles.]

$$2(3 + 5x) = 6 + 5x$$

4-18. Find an equation for the line passing through the points $(-3, 1)$ and $(9, 7)$.
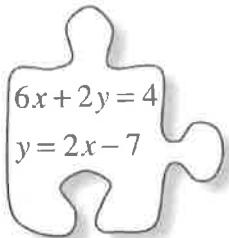[$y = \frac{1}{2}x + \frac{5}{2}$]

4-19. Multiply each polynomial. That is, change each product to a sum.

a. $(2x + 1)(3x - 2)$
[$6x^2 - x - 2$]

b. $(2x + 1)(3x^2 - 2x - 5)$
[$6x^3 - x^2 - 12x - 5$]

4.1.2 How many equations do I need?

$$6x + 2y = 4$$
$$y = 2x - 7$$

One Equation or Two?

In the previous lesson, you created one or two mathematical sentences that represented word problems. Today you will represent a word problem with two equations. You will also explore how to use the Equal Values Method to solve systems containing equations that are not in $y = mx + b$ form.

4-20.   ONE EQUATION OR TWO?

Elsie took all of her cans and bottles from home to the recycling plant. The number of cans was one more than four times the number of bottles. She earned 12¢ for each bottle and 10¢ for each can, and ended up earning $2.18 in all. How many cans and bottles did she recycle?

Solomon decided to solve the problem by writing one equation. He said, *"I can let b represent the number of bottles. Then $4b + 1$ would be the number of cans. My equation would be $12b + 10(4b + 1) = 218$."*

Marcus agreed with Solomon's answer, but said, *"It is easier to solve this problem with two equations. I can let b represent the number of bottles and c represent the number of cans. That way my two equations are $c = 4b + 1$ and $12b + 10c = 218$."*

a.   Solomon's equation has three terms: $12b$, $10(4b + 1)$, and 218. What do each of these terms represent in the problem? [**$12b$ represents 12 cents per bottle and the number of bottles. 10 represents 10 cents per can and $4b + 1$ represents that there are one more than four times the number of cans than bottles. 218 represents the $2.18 which is the total amount of money that Elsie made.**]

b.   What do the parts of each of Marcus' equations represent? [**$c = 4b + 1$ represents that the number of cans is one more than four times the number of bottles. $12b + 10c = 218$ represents that Elsie made 12 cents per bottle plus 10 cents per can for a total of $2.18.**]

c.   Do Solomon's equation and Marcus' equations represent the same problem? Why or why not? [**Yes, because they both represent the price of can plus the price of bottles equals the total amount of money. They also both accurately represent the number of cans versus bottles.**]

d.   Solve this problem using Solomon's equation. Be sure to label your answer. You do not need to solve Marcus' equations. [**17 cans and 4 bottles**]

4-21.   Renard thought that writing two equations for problem 4-20 was easy, but he's not sure if he knows how to solve the system of equations. He wants to use two equations with two variables to solve this problem:

> Ariel bought several bags of caramel candy and taffy. The number of bags of taffy was 5 more than the number of bags of caramels. Taffy bags weigh 8 ounces each, and caramel bags weigh 16 ounces each. The total weight of all of the bags of candy was 400 ounces. How many bags of candy did she buy?

a.   Renard lets t = the number of taffy bags and c = the number of caramel bags. Help him write two equations to represent the information in the problem. [$8t + 16c = 400$, $t = 5 + c$]

b.   Now Renard is stuck. He says, "If both of the equations were in the form 't = something,' I could set the two equations equal to each other to find the solution." Help him change the equations into a form he can solve. [$t = 50 - 2c$, $t = 5 + c$]

c.   Solve Renard's equations to find the number of caramel and taffy bags that Ariel bought. [$t = 20$, $c = 15$]

d.   Discuss with your team how you can make sure your solution is correct. [**Substitute t and c back into the *original two* equations.**]

4-22.   When you write equations to solve word problems, you sometimes end up with two equations like Renard's, or like the two equations at right. Notice that the second equation is solved for y, but the first is not. Change the first equation into "$y =$" form, and then solve this system of equations. Check your solution. [**(6, 1)**]

$$x - 2y = 4$$
$$y = -\tfrac{1}{2}x + 4$$

4-23.   A set of two or more equations with the same variables is called a **system of equations**. When you set the two equations equal to each other, like Renard did in problem 4-21, you are using the **Equal Values Method** of solving a system of equations.

There are 21 animals on Farmer Cole's farm – all sheep and chickens. If the animals have a total of 56 legs, how many of each type of animal lives on his farm? Write a system of equations, and use the Equal Values Method to solve it. Be sure to check your answer. [**Let s = the number of sheep, and let c = the number of chickens, then $c + s = 21$ and $2c + 4s = 56$. Using the Equal Values Method, $21 - s = 28 - 2s$ or $21 - c = 14 - 0.5c$; $s = 7$ and $c = 14$. Students should check their answers in the *original two* equations.**]

4-24.   Solve the system of equations at right using the Equal Values $x + 2y = 14$
          Method. Check your answer. [(**–2, 8**)] $-x + 3y = 26$

METHODS AND MEANINGS

The Equal Values Method

The **Equal Values Method** is a method to find the solution to a system of equations. For example, solve the system of equations below:

$$2x + y = 5$$
$$y = x - 1$$

Put both equations into $y = mx + b$ form.
The two equations are now $y = -2x + 5$ and
$y = x - 1$.

$2x + y = 5$	$y = x - 1$
$y = -2x + 5$	

Take the two expressions that equal y and set
them equal to each other. Then solve this new
equation to find x. See the example at right.

$$-2x + 5 = x - 1$$
$$-3x = -6$$
$$x = 2$$

Once you know x, substitute your solution for x
into *either* original equation to find y. In this
example, the second equation is used.

$$y = x - 1$$
$$y = (2) - 1$$
$$y = 1$$

Check your solution by evaluating for x and y
in *both* of the *original* equations.

|	
$2x + y = 5$	$y = x - 1$
$2(2) + 1 = 5$	$1 = 2 - 1$
$5 = 5$	$1 = 1$
✓	✓

The solution is $x = 2$ and $y = 1$.

4-25.   Write expressions to represent the quantities described below.

a.   Geraldine is 4 years younger than Tom. If Tom is t years old, how old is Geraldine? Also, if Steven is twice as old as Geraldine, how old is he? [$t-4$; $2(t-4)$]

b.   150 people went to see "Ode to Algebra" performed in the school auditorium. If the number of children that attended the performance was c, how many adults attended? [$150-c$]

c.   The cost of a new CD is $14.95, and the cost of a video game is $39.99. How much would c CDs and v video games cost? [$14.95c+39.99v$]

4-26.   Nina has some nickels and 9 pennies in her pocket. Her friend, Maurice, has twice as many nickels as Nina. Together, these coins are worth 84¢. How many nickels does Nina have? Show all of your work and label your answers. [**If Nina has n nickels, then $5n+9+5(2n)=84$, and $n = 5$ nickels.**]

4-27.   Create a table and graph the equation $y = 10 - x^2 + 3x$. Label its x- and y-intercepts. [**See table below and graph at right. x-intercepts (–2, 0) and (5, 0) and y-intercept (0, 10)**]

x	–3	–2	–1	0	1	2	3	4	5	6
y	–8	0	6	10	12	12	10	6	0	–8
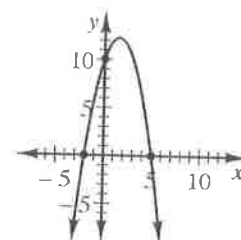
4-28.   Examine the graphs of each relation below. Decide if each is a function. Then describe the domain and range of each one. [**a: not a function, D: $-3 \le x \le 3$ and R: $-3 \le y \le 3$; b: a function, D: $-2 \le x \le 3$, R: $-2 \le x \le 2$**]

a.

b.
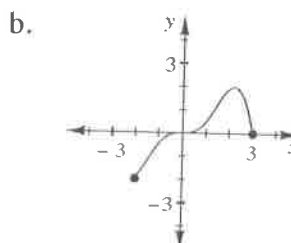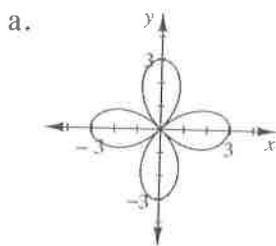
4-29.   What number is not part of the domain of the function $g(x) = \frac{x+2}{x-1}$? How can you tell? [$x = 1$; **It will create a fraction with a denominator of zero, which is undefined.**]

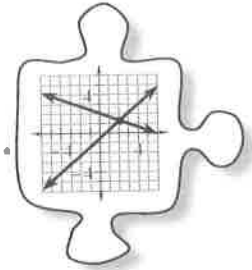4-30.   If $f(x) = 3x - 9$ and $g(x) = -x^2$, find:

a. $f(-2)$ b. $g(-2)$ c. x if $f(x) = 0$ d. $g(m)$
       [−15]   [−4]   [3]   [$-m^2$]

4.2.1 How can I solve the system?

Solving Systems of Equations Using Substitution

In Lesson 4.1.2, you helped Renard develop the Equal Values Method of solving a system of equations. You set both of the equations equal to the same variable. Today you will develop a more efficient method of solving systems that are too messy to solve by setting the equations equal to each other.

4-31. Review what you learned in Lesson 4.1.2 as you solve the system of equations below. Check your solution. **[The solution is (–11, 4), although some students may not find the solution by the time you move the class on to problem 4-32.]**

$$y = -x - 7$$
$$5y + 3x = -13$$

4-32. AVOIDING THE MESS

A new method, called the **Substitution Method**, can help you solve the system in problem 4-31 without using fractions. This method is outlined below.

a. If $y = -x - 7$, then does $-x - 7 = y$? That is, can you switch the y and the $-x - 7$? Why or why not? **[Yes; the two quantities are equal.]**

$$\overset{\frown}{y} = \overset{\frown}{(-x - 7)}$$
$$5y + 3x = -13$$

b. Since you know that $y = -x - 7$, can you replace the y in the second equation with $-x - 7$ from the top equation? Why or why not? **[Yes; again, we can switch these values because the top equation indicates that they are equal.]**

$$y = (-x - 7)$$
$$5(y) + 3x = -13$$

c. Once you replace the y in the second equation with $-x - 7$, you have an equation with only one variable, as shown below. This is called **substitution** because you are substituting for (replacing) y with an expression that it equals. Solve this new equation for x and then use that result to find y in either of the original equations. **[$x = -11$, $y = 4$]**

$$5(-x - 7) + 3x = -13$$

4-33.   Use the Substitution Method to solve the systems of equations below.

a. $y = 3x$ [$x = 4, y = 12$]
       $2y - 5x = 4$

b. $x - 4 = y$ [$x = 3, y = -1$]
       $-5y + 8x = 29$

c. $2x + 2y = 18$ [no solution]
       $x = 3 - y$

d. $c = -b - 11$ [$b = -3, c = -8$]
       $3c + 6 = 6b$

4-34.   When Mei solved the system of equations below, she got the solution $x = 4$, $y = 2$. *Without solving the system yourself*, can you tell her whether this solution is correct? How do you know? [**Yes, she is correct. To test, substitute the values for x and y into both equations to see if they are correct solutions.**]

$$4x + 3y = 22$$
$$x - 2y = 0$$

4-35.   HAPPY BIRTHDAY!

You've decided to give your best friend a bag of marbles for his birthday. Since you know that your friend likes green marbles better than red ones, the bag has twice as many green marbles as red. The label on the bag says it contains a total of 84 marbles.

How many green marbles are in the bag? Write an equation (or system of equations) for this problem. Then solve the problem using any method you choose. Be sure to check your answer when you are finished.
[**There are 28 red and 56 green marbles.**]

4-36. Ms. Hoang's class conducted an experiment by rolling a marble down different lengths of slanted boards and timing how long it took. The results are shown below. Describe the association. Refer to the Math Notes box in this lesson if you need help remembering how to describe an association. **[A very strong positive non-linear association with no apparent outliers.]**
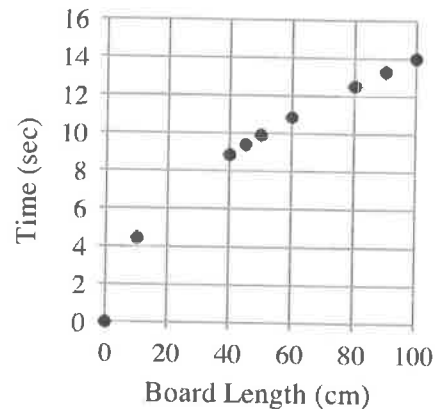
4-37. Solve each equation for the variable. Check your solutions, if possible.

a. $8a + a - 3 = 6a - 2a - 3$

 [$a = 0$]

b. $(m+2)(m+3) = (m+2)(m-2)$

 [$m = -2$]

c. $\frac{x}{2} + 1 = 6$

 [$x = 10$]

d. $4t - 2 + t^2 = 6 + t^2$

 [$t = 2$]

4-38. The Fabulous Footballers scored an incredible 55 points at last night's game. Interestingly, the number of field goals was 1 more than twice the number of touchdowns. The Fabulous Footballers earned 7 points for each touchdown and 3 points for each field goal.

a. **Multiple Choice:** Which system of equations below best represents this situation? Explain your reasoning. Assume that t represents the number of touchdowns and f represents the number of field goals. **[ii]**

i. $t = 2f + 1$
    $7t + 3f = 55$

ii. $f = 2t + 1$
     $7t + 3f = 55$

iii. $t = 2f + 1$
      $3t + 7f = 55$

iv. $f = 2t + 1$
     $3t + 7f = 55$

b. Solve the system you selected in part (a) and determine how many touchdowns and field goals the Fabulous Footballers earned last night. **[4 touchdowns and 9 field goals]**

4-39. Yesterday Mica was given some information and was asked to find a linear equation. But last night her cat destroyed most of the information! At right is all she has left:
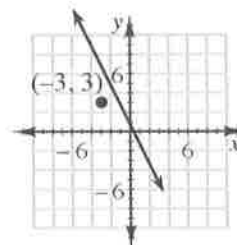
a. Complete the table and graph the line that represents Mica's equation. [**See answers in bold in table and line on graph.**]

x	y
-3	[**3**]
-2	1
-1	[**-1**]
0	[**-3**]
1	[**-5**]
2	[**-7**]
3	[**-9**]

b. Mica thinks the equation for this graph could be $2x + y = -3$. Is she correct? Explain why or why not. If not, find your own algebraic equation to match the graph and $x \rightarrow y$ table. [**Yes; (-3, 3) and (-2, 1) both make this equation true.**]

4-40. Kevin and his little sister, Katy, are trying to solve the system of equations shown below. Kevin thinks the new equation should be $3(6x - 1) + 2y = 43$, while Katy thinks it should be $3x + 2(6x - 1) = 43$. Who is correct and why? [**Katy is correct; the $6x - 1$ should be substituted for y because they are equal.**]

$$y = 6x - 1$$
$$3x + 2y = 43$$

4-41. Simplify each expression. In parts (c) and (d) write your answers using scientific notation.

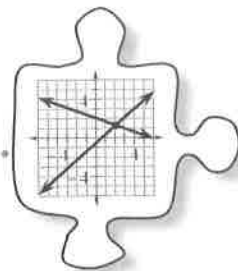a. $5^0 \cdot 2^{-3}$ [$\frac{1}{8}$]

b. $\frac{a^3}{b^{-2}} \cdot \frac{ab^2}{a^4}$ [b^4]

c. $2.3 \times 10^{-3} \cdot 4.2 \times 10^2$ [9.66×10^{-1}]

d. $(3.5 \times 10^3)^2$ [1.225×10^7]

4.2.2 How does a graph show a solution?

. . . .

Making Connections: Systems, Solutions, and Graphs

In this chapter you have practiced writing mathematical sentences to represent situations. Often, these sentences give you a system of equations, which you can solve using substitution. Today you will also represent these situations on a graph and will examine more closely the solution to a two-variable equation.

4-42.   THE HILLS ARE ALIVE

The Alpine Music Club is going on its annual music trip. The members of the club are yodelers, and they like to play the xylophone. This year they are taking their xylophones on a gondola to give a performance at the top of Mount Monch.

The gondola conductor charges $2 for each yodeler and $1 for each xylophone. It costs $40 for the entire club, including the xylophones, to ride the gondola. Two yodelers can share a xylophone, so the number of yodelers on the gondola is twice the number of xylophones.

How many yodelers and how many xylophones are on the gondola? **[8 xylophones and 16 yodelers]**

Your Task:

- Represent this problem with a system of equations. Solve the system and explain how its solution relates to the yodelers on the music trip.

- Represent this problem with a graph. Identify how the solution to this problem appears on the graph.

Discussion Points

How can the given information be represented with equations?

What is a solution to a two-variable equation?

How can this problem be represented on a graph?

How does the solution appear on the graph?

Further Guidance

4-43.   Start by focusing on one aspect of the problem: the cost to ride the gondola. The conductor charges $2 for each yodeler and $1 for each xylophone. It costs $40 for the entire club, with instruments, to ride the gondola.

a.   Write an equation with two variables that represents this information. Be sure to define your variables. **[Let x = the number of xylophones and y = the number of yodelers; $2y + 1x = 40$.]**

b.   Find a combination of xylophones and yodelers that will make your equation from part (a) true. Is this is the only possible combination? **[Answers vary but could include: 10 yodelers and 20 xylophones, 15 yodelers and 10 xylophones.]**

c.   List five additional combinations of xylophones and yodelers that could ride the gondola if it costs $40 for the trip. With your team, decide on a good way to organize and share your list. **[The information could be organized in an $x \to y$ table or as a list of ordered pairs. The combinations could include: $(20, 10), (10, 15), (4, 18), (30, 5), (12, 14), (14, 13)$, etc.]**

d.   Jon says, "I think there could be 28 xylophones and 8 yodelers on the gondola." Is he correct? Use the equation you have written to explain why or why not. **[No; $2(8) + 28 \neq 40$]**

e.   Helga says, "Each correct combination we found is a *solution* to our equation." Is this true? Explain what it means for something to be a solution to a two-variable equation. **[Yes; a solution makes the equation true.]**

4-44.   Now consider the other piece of information: The number of yodelers is twice the number of xylophones.

a.   Write an equation (mathematical sentence) that expresses this piece of information. **[$y = 2x$]**

b.   List four different combinations of xylophones and yodelers that will make this equation true. **[$(1, 2), (2, 4), (3, 6), (4, 8)$, etc.]**

c.   Put the equation you found in part (a) together with your equation from problem 4-43 and use substitution to solve this system of equations. **[$(8, 16)$]**

d.   Is the answer you found in part (c) a solution to the first equation you wrote (the equation in part (a) of problem 4-43)? How can you check? Is it a solution to the second equation you wrote (the equation in part (a) of this problem)? Why is this a solution to the *system* of equations? **[It makes both equations true.]**

4-45.   The solution to "The Hills are Alive" problem can also be represented graphically.

a.   On graph paper, graph the equation you wrote in part (a) of problem 4-43. The points you listed for that equation may help. What is the shape of this graph? Label your graph with its equation. [**A line; $2y+x=40$ or $y=-\frac{1}{2}x+20$**]

b.   Explain how each point on the graph represents a solution to the equation. [**When each point on the line is substituted into the equation, it makes the equation true.**]

c.   Now graph the equation you wrote in part (a) of problem 4-44 on the same set of axes. The points you listed for that equation may help. Label this graph with its equation. [**Students should graph the line $y=2x$.**]

d.   Find the intersection point of the two graphs. What is special about this point? [**(8, 16); It makes both equations true.**]

e.   With your team, find as many ways as you can to express the solution to "The Hills are Alive" problem. Be prepared to share all the different forms you found for the solution with the class. [**Answers vary. Common methods: as a point (x, y), as a statement (such as "$x = \ldots$ and $y = \ldots$"), or as a sentence (such as, "The club had 16 yodelers and 8 xylophones.")**]

_____   *Further Guidance*   _____
                       section ends here.

4-46.   Consider this system of equations:

$$2x + 2y = 18$$
$$y = x - 3$$

a.   Use substitution to solve this system. [**(6, 3)**]

b.   With your team, decide how to fill in the rest of the table at right for the equation $2x+2y=18$. [**See answers in bold in the table.**]

c.   Use your table to make an accurate graph of the equation $2x+2y=18$. [**The line should have a y-intercept of (0, 9) and slope of –1.**]

x	y
–2	11
–1	[**10**]
0	[**9**]
1	[**8**]
2	[**7**]
3	[**6**]

d.   Now graph $y = x - 3$ on the same set of axes. Find the point of intersection. [**(6, 3)**]

e.   Does the point of intersection you found in part (a) agree with what you see on your graph? [**Yes**]

4-47.   The equations of two lines are given below. A table of solutions for the first equation has been started below the equation.

$$10x + 4y = -8 \qquad y = -\tfrac{5}{2}x - 2$$

x	y
-4	8
-2	
0	
2	

a.   Graph both lines. Without actually solving the system of equations, predict what the solution to this system will be. Explain. [**There will be infinite solutions. The lines coincide, so any point that is a solution for the first equation is also a solution for the second equation.**]

b.   Solve the system. Was your prediction in part (a) correct? [**There are infinite solutions.**]

4-48.   What is a solution to a two-variable equation? Answer this question in complete sentences in your Learning Log. Then give an example of a two-variable equation followed by two different solutions to it. Finally, make a list of all of the ways to represent solutions to two-variable equations. Title your entry "Solutions to Two-Variable Equations" and label it with today's date.

4-50. The graph at right contains the lines for $y = x + 2$ and $y = 2x - 1$.

    a. Using the graph, what is the solution to this system? [**(3, 5)**]

   b. Solve the system algebraically to confirm your answer to part (a).
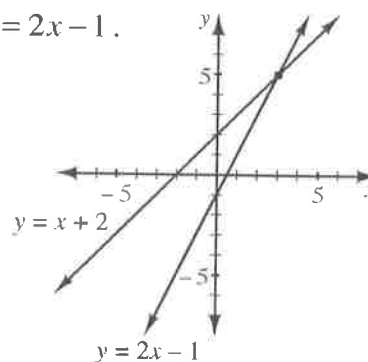
4-51. Hotdogs and corndogs were sold at last night's football game. Use the information below to write mathematical sentences to help you determine how many corndogs were sold.

    a. The number of hotdogs sold was three fewer than twice the number of corndogs. Write a mathematical sentence that relates the number of hotdogs and corndogs. Let h represent the number of hotdogs and c represent the number of corndogs. [$h = 2c - 3$]

    b. A hotdog costs $3 and a corndog costs $1.50. If $201 was collected, write a mathematical sentence to represent this information. [$3h + 1.5c = 201$]

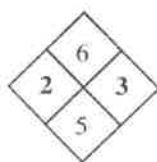   c. How many corndogs were sold? Show how you found your answer. [**28 corndogs were sold.**]

4-52. Copy and complete each of the Diamond Problems below. The pattern used in the Diamond Problems is shown at right. [**Find solutions in the diamonds below.**]
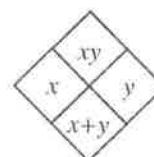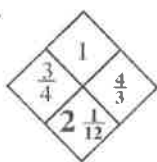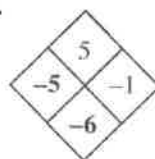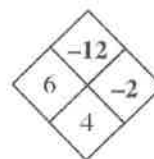
    a.   b.   c.   d.

4-53. Rianna thinks that if $a = b$ and if $c = d$, then $a + c = b + d$. Is she correct? [**Yes; adding equal values to both sides of an equality preserves the equality.**]

4-54. Solve the following equations for x, if possible. Check your solutions.

    a. $-(2 - 3x) + x = 9 - x$ [$x = 2.2$]   b. $\frac{6}{x+2} = \frac{3}{4}$ [$x = 6$]

    c. $5 - 2(x + 6) = 14$ [$x = -10.5$]   d. $\frac{1}{2}x - 4 + 1 = -3 - \frac{1}{2}x$ [$x = 0$]