Danica wants to arrange her model cars in rows with exactly 6 cars in each row. She now has 23 model cars. What is the smallest number of additional cars she must buy in order to be able to arrange her cars in this way?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

2013 AMC 8, Problem #1— "What is the smallest multiple of 6 that is at least 24?"

Solution

Answer (A): The smallest multiple of 6 that is at least 23 is 24, so Danica must buy 1 additional car.

Difficulty: Easy
SMP-CCSS: 1. Make sense of problems and persevere in solving them; 7. Look for and make use of structure.
CCSS-M: 6.NS.B. Compute fluently with multi-digit numbers and find common factors and multiples.

What is the value of $4 \cdot (-1 + 2 - 3 + 4 - 5 + 6 - 7 + \dots + 1000)$?

(A) -10 (B) 0 (C) 1 (D) 500 (E) 2000

2013 AMC 8, Problem #3— "Find the sum inside the parentheses."

Solution

Answer (E): Inside the parentheses are 500 pairs of numbers, each with a sum of 1. Therefore the expression equals $4 \cdot 500 = 2000$.

Difficulty: Medium

SMP-CCSS: 1. Make sense of problems and persevere in solving them; 7. Look for and make use of structure.

What is the ratio of the least common multiple of 180 and 594 to the greatest common factor of 180 and 594?

(A) 110 (B) 165 (C) 330 (D) 625 (E) 660

2013 AMC 8, Problem #10— "Find the prime factorization of 180 and 594."

Solution

Answer (C): Because the prime factorizations of 180 and 594 are $2^2 \cdot 3^2 \cdot 5$ and $2 \cdot 3^3 \cdot 11$, respectively, the least common multiple of 180 and 594 is $2^2 \cdot 3^3 \cdot 5 \cdot 11$, and their greatest common factor is $2 \cdot 3^2$. The ratio of their least common multiple to their greatest common factor is $\frac{2^2 \cdot 3^3 \cdot 5 \cdot 11}{2 \cdot 3^2} = 2 \cdot 3 \cdot 5 \cdot 11 = 330$.

Difficulty: Medium HardSMP-CCSS: 1. Make sense of problems and persevere in solving them.CCSS-M: 6.NS.B. Compute fluently with multi-digit numbers and find common factors and multiples.

A 1×2 rectangle is inscribed in a semicircle with the longer side on the diameter. What is the area of the semicircle?

(A)
$$\frac{\pi}{2}$$
 (B) $\frac{2\pi}{3}$ (C) π (D) $\frac{4\pi}{3}$ (E) $\frac{5\pi}{3}$

2013 AMC 8, Problem #20—

"Use the Pythagorean Theorem to find the radius of the semicircle."



Answer (C):



By the Pythagorean Theorem, the radius of the semicircle is $\sqrt{2}$, so its area is $\frac{\pi(\sqrt{2})^2}{2} = \pi$.

Difficulty: HardSMP-CCSS: 1. Make sense of problems and persevere in solving them.CCSS-M: 8.G.B. Understand and apply the Pythagorean Theorem.

Angle ABC of $\triangle ABC$ is a right angle. The sides of $\triangle ABC$ are the diameters of semicircles as shown. The area of the semicircle on \overline{AB} equals 8π , and the arc of the semicircle on \overline{AC} has length 8.5π . What is the radius of the semicircle on \overline{BC} ?



(A) 7 (B) 7.5 (C) 8 (D) 8.5 (E) 9

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2013 AMC 8, Problem #23-

"Find the area for the circle with diameter \overline{AB} ."

Solution

Answer (B): The circle with diameter \overline{AB} has twice the area of the corresponding semicircle; thus the area of the circle is 16π and its radius is 4. Consequently AB = 8. The circle with diameter \overline{AC} has circumference 17π , so AC = 17. \overline{AC} is the hypotenuse of the right triangle. By the Pythagorean Theorem, $17^2 = 8^2 + (BC)^2$. Therefore BC = 15, and the radius is 7.5.

Difficulty: Medium HardSMP-CCSS: 1. Make sense of problems and persevere in solving them.CCSS-M: 8.G.B. Understand and apply the Pythagorean Theorem.

Toothpicks are used to make a grid that is 60 toothpicks long and 32 toothpicks high. How many toothpicks are used altogether?



(A) 1920 (B) 1952 (C) 1980 (D) 2013 (E) 3932

2013 AMC 8, Problem #22-

"Find the number of columns and rows for 60 toothpicks long and 32 toothpicks high."

Solution

Answer (E): A grid 60 toothpicks long and 32 toothpicks high needs 61 columns of 32 toothpicks and 33 rows of 60 toothpicks. Therefore a total of $(61 \times 32) + (33 \times 60) = 3932$ toothpicks are needed.

Difficulty: Hard SMP-CCSS: 1. Make sense of problems and persevere in solving them; 7. Look for and make use of structure. CCSS-M: None

A fair coin is tossed 3 times. What is the probability of at least two consecutive heads?

(A)
$$\frac{1}{8}$$
 (B) $\frac{1}{4}$ (C) $\frac{3}{8}$ (D) $\frac{1}{2}$ (E) $\frac{3}{4}$

2013 AMC 8, Problem #8— "List the number of possible equally likely outcomes."

Solution

Answer (C): List the 8 possible equally likely outcomes: HHH, HHT, HTH, HTT, THH, THT, TTH, TTT. Only HHH, HHT, THH have at least 2 consecutive heads, so the probability of at least 2 consecutive heads is $\frac{3}{8}$.

Difficulty: Medium Hard

SMP-CCSS: 1. Make sense of problems and persevere in solving them; 4. Model with mathematics. **CCSS-M:** 7.SP.C. Investigate chance processes and develop, use, and evaluate probability models.

A sign at the fish market says, "50% off, today only: halfpound packages for just \$3 per package." What is the regular price for a full pound of fish, in dollars?

(A) 6 (B) 9 (C) 10 (D) 12 (E) 15

2013 AMC 8, Problem #2—

"How much does a half-pound package cost at the regular price?"

Solution

Answer (D): A half-pound package costs \$3 at the sale price, so it would cost \$6 at the regular price. A whole pound would cost \$12 at the regular price.

Difficulty: Medium Easy

SMP-CCSS: 1. Make sense of problems and persevere in solving them; 4. Model with mathematics. CCSS-M: 7.RP.A. Analyze proportional relationships and use them to solve real-world and mathematical problems.