A

MULTIPLICATION OF FRACTIONS

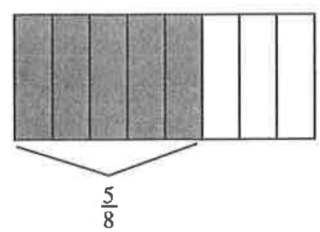
MULTIPLYING FRACTIONS WITH AN AREA MODEL

Multiplication of fractions is reviewed using a rectangular area model. Lines that divide the rectangle to represent one fraction are drawn vertically, and the correct number of parts are shaded. Then lines that divide the rectangle to represent the second fraction are drawn horizontally and part of the shaded region is darkened to represent the product of the two fractions.

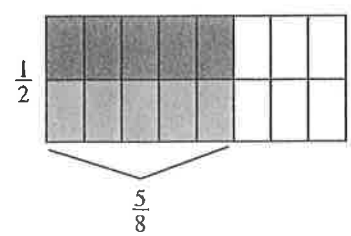
Example 1

$\frac{1}{2} \cdot \frac{5}{8}$ (that is, $\frac{1}{2}$ of $\frac{5}{8}$)

Step 1: Draw a generic rectangle and divide it into 8 pieces vertically. Lightly shade 5 of those pieces. Label it $\frac{5}{8}$.



Step 2: Use a horizontal line and divide the generic rectangle in half. Darkly shade $\frac{1}{2}$ of $\frac{5}{8}$ and label it.



Step 3: Write a number sentence. $\frac{1}{2} \cdot \frac{5}{8} = \frac{5}{16}$

The rule for multiplying fractions derived from the models above is to multiply the numerators, then multiply the denominators. Simplify the product when possible.

For additional information, see the Math Notes boxes in Lesson 5.1.4 of the *Core Connections, Course 1* text or Lesson 2.2.5 of the *Core Connections, Course 2* text. For additional examples and practice, see the *Core Connections, Course 1* Checkpoint 7A materials.

Example 2

a. $\frac{2}{3}\cdot\frac{2}{7} \Rightarrow \frac{2\cdot 2}{3\cdot 7} \Rightarrow \frac{4}{21}$

b. $\frac{3}{4}\cdot\frac{6}{7} \Rightarrow \frac{3\cdot 6}{4\cdot 7} \Rightarrow \frac{18}{28} \Rightarrow \frac{9}{14}$

Problems

Draw an area model for each of the following multiplication problems and write the answer.

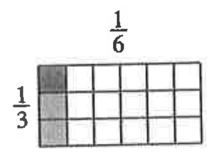
1. $\frac{1}{3}\cdot\frac{1}{6}$
2. $\frac{1}{4}\cdot\frac{3}{5}$
3. $\frac{2}{3}\cdot\frac{5}{9}$

Use the rule for multiplying fractions to find the answer for the following problems. Simplify when possible.

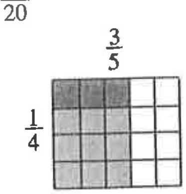
4. $\frac{1}{3}\cdot\frac{2}{5}$
5. $\frac{2}{3}\cdot\frac{2}{7}$
6. $\frac{3}{4}\cdot\frac{1}{5}$
7. $\frac{2}{5}\cdot\frac{2}{3}$
8. $\frac{2}{3}\cdot\frac{1}{4}$
9. $\frac{5}{6}\cdot\frac{2}{3}$
10. $\frac{4}{5}\cdot\frac{3}{4}$
11. $\frac{2}{15}\cdot\frac{1}{2}$
12. $\frac{3}{7}\cdot\frac{1}{2}$
13. $\frac{3}{8}\cdot\frac{4}{5}$
14. $\frac{2}{9}\cdot\frac{3}{5}$
15. $\frac{3}{10}\cdot\frac{5}{7}$
16. $\frac{5}{11}\cdot\frac{6}{7}$
17. $\frac{5}{6}\cdot\frac{3}{10}$
18. $\frac{10}{11}\cdot\frac{3}{5}$
19. $\frac{5}{12}\cdot\frac{3}{5}$
20. $\frac{7}{9}\cdot\frac{5}{14}$

Answers

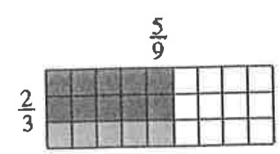
1. $\frac{1}{18}$



2. $\frac{3}{20}$



3. $\frac{10}{27}$



4. $\frac{2}{15}$
5. $\frac{4}{21}$
6. $\frac{3}{20}$
7. $\frac{4}{15}$
8. $\frac{2}{12}=\frac{1}{6}$
9. $\frac{10}{18}=\frac{5}{9}$
10. $\frac{12}{20}=\frac{3}{5}$
11. $\frac{2}{30}=\frac{1}{5}$
12. $\frac{3}{14}$
13. $\frac{12}{40}=\frac{3}{10}$
14. $\frac{6}{45}=\frac{2}{15}$
15. $\frac{15}{70}=\frac{3}{14}$
16. $\frac{30}{77}$
17. $\frac{15}{60}=\frac{1}{4}$
18. $\frac{30}{55}=\frac{6}{11}$
19. $\frac{15}{60}=\frac{1}{4}$
20. $\frac{35}{126}=\frac{5}{18}$

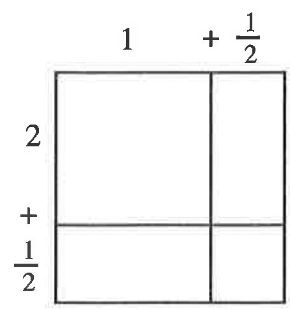
MULTIPLYING MIXED NUMBERS

There are two ways to multiply mixed numbers. One is with generic rectangles. For additional information, see the Math Notes box in Lesson 2.3.1 of the *Core Connections, Course 2* text.

Example 1

Find the product: $2\frac{1}{2} \cdot 1\frac{1}{2}$.

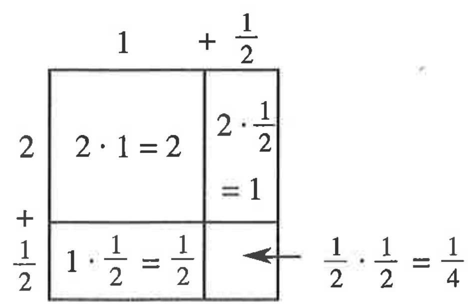
Step 1: Draw the generic rectangle. Label the top 1 plus $\frac{1}{2}$. Label the side 2 plus $\frac{1}{2}$.



Step 2: Write the area of each smaller rectangle in each of the four parts of the drawing.

Find the total area:

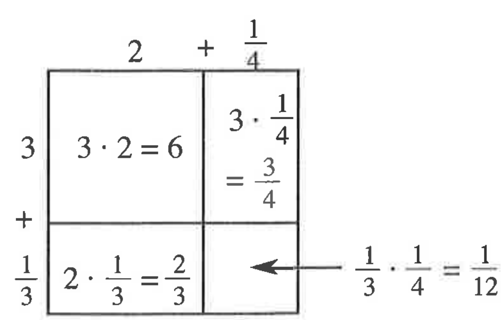
$2 + 1 + \frac{1}{2} + \frac{1}{4} = 3\frac{3}{4}$



Step 3: Write a number sentence: $2\frac{1}{2} \cdot 1\frac{1}{2} = 3\frac{3}{4}$

Example 2

Find the product: $3\frac{1}{3} \cdot 2\frac{1}{4}$.



$$6 + \frac{3}{4} + \frac{2}{3} + \frac{1}{12} \Rightarrow 6 + \frac{9}{12} + \frac{8}{12} + \frac{1}{12} \Rightarrow$$
$$\Rightarrow 6\frac{18}{12} \Rightarrow 7\frac{1}{2}$$

Problems

Use a generic rectangle to find each product.

1. $1\frac{1}{4} \cdot 1\frac{1}{2}$
2. $3\frac{1}{6} \cdot 2\frac{1}{2}$
3. $2\frac{1}{4} \cdot 1\frac{1}{2}$
4. $1\frac{1}{3} \cdot 1\frac{1}{6}$
5. $1\frac{1}{2} \cdot 1\frac{1}{3}$

Answers

1. $1\frac{7}{8}$

	1	$+\ \frac{1}{2}$
1	1	$\frac{1}{2}$
$+\ \frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{8}$

2. $7\frac{11}{12}$

	2	$+\ \frac{1}{2}$
3	6	$\frac{3}{2}$
$+\ \frac{1}{6}$	$\frac{2}{6}$	$\frac{1}{12}$

3. $3\frac{3}{8}$

	1	$+\ \frac{1}{2}$
2	2	1
$+\ \frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{8}$

4. $1\frac{5}{9}$

	1	$+\ \frac{1}{6}$
1	1	$\frac{1}{6}$
$+\ \frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{18}$

5. 2

	1	$+\ \frac{1}{3}$
1	1	$\frac{1}{3}$
$+\ \frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{6}$

You can also multiply mixed numbers by changing them to fractions greater than 1, then multiplying the numerators and multiplying the denominators. Simplify if possible.

For more information, see the Math Notes box in Lesson 5.2.1 of the *Core Connections, Course 1* text. For additional examples and practice, see the *Core Connections, Course 1* Checkpoint 7A materials.

Example 3

$$2\tfrac{1}{2}\cdot 1\tfrac{1}{4} \Rightarrow \tfrac{5}{2}\cdot\tfrac{5}{4} \Rightarrow \tfrac{5\cdot 5}{2\cdot 4} \Rightarrow \tfrac{25}{8} \Rightarrow 3\tfrac{1}{8}$$

Problems

Find each product, using the method of your choice. Simplify when possible.

1. $2\frac{1}{4}\cdot 1\frac{3}{8}$
2. $3\frac{3}{5}\cdot 2\frac{4}{7}$
3. $2\frac{3}{8}\cdot 1\frac{1}{6}$
4. $3\frac{7}{9}\cdot 2\frac{5}{8}$
5. $1\frac{2}{9}\cdot 2\frac{3}{7}$
6. $3\frac{4}{7}\cdot 5\frac{8}{11}$
7. $2\frac{3}{8}\cdot 1\frac{1}{16}$
8. $2\frac{8}{9}\cdot 2\frac{5}{8}$
9. $1\frac{1}{3}\cdot 1\frac{4}{7}$
10. $2\frac{1}{7}\cdot 2\frac{7}{10}$

Answers

1. $3\frac{3}{32}$
2. $9\frac{9}{35}$
3. $2\frac{37}{48}$
4. $9\frac{11}{12}$
5. $2\frac{61}{63}$
6. $20\frac{5}{11}$
7. $2\frac{67}{128}$
8. $7\frac{7}{12}$
9. $2\frac{2}{21}$
10. $5\frac{11}{14}$

DIVISION BY FRACTIONS

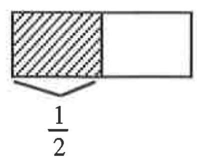
Division by fractions introduces three methods to help students understand how dividing by fractions works. In general, think of division for a problem like $8 \div 2$ as, "In 8, how many groups of 2 are there?" Similarly, $\frac{1}{2} \div \frac{1}{4}$ means, "In $\frac{1}{2}$, how many fourths are there?"

For more information, see the Math Notes boxes in Lessons 7.2.2 and 7.2.4 of the *Core Connections, Course 1* text or Lesson 3.3.1 of the *Core Connections, Course 2* text. For additional examples and practice, see the *Core Connections, Course 1* Checkpoint 8B materials. The first two examples show how to divide fractions using a diagram.

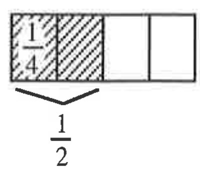
Example 1

Use the rectangular model to divide: $\frac{1}{2} \div \frac{1}{4}$.

Step 1: Using the rectangle, we first divide it into 2 equal pieces. Each piece represents $\frac{1}{2}$. Shade $\frac{1}{2}$ of it.



Step 2: Then divide the *original* rectangle into four equal pieces. Each section represents $\frac{1}{4}$. In the shaded section, $\frac{1}{2}$, there are 2 fourths.



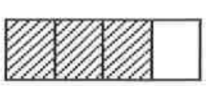
Step 3: Write the equation.

$\frac{1}{2} \div \frac{1}{4} = 2$

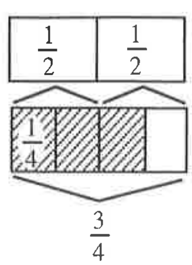
Example 2

In $\frac{3}{4}$, how many $\frac{1}{2}$'s are there?

That is, $\frac{3}{4} \div \frac{1}{2} = ?$



Start with $\frac{3}{4}$.



In $\frac{3}{4}$ there is one full $\frac{1}{2}$ shaded and half of another one (that is half of one-half).

So: $\frac{3}{4} \div \frac{1}{2} = 1\frac{1}{2}$
(one and one-half halves)

Problems

Use the rectangular model to divide.

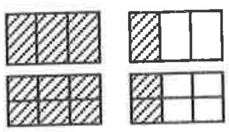
1. $1\frac{1}{3} \div \frac{1}{6}$ 2. $\frac{3}{2} \div \frac{3}{4}$ 3. $1 \div \frac{1}{4}$ 4. $1\frac{1}{4} \div \frac{1}{2}$ 5. $2\frac{2}{3} \div \frac{1}{9}$

Answers

1. 8

3rds

6ths

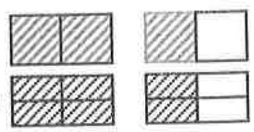


8 sixths

2. 2

halves

quarters

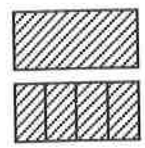


2 three fourths

3. 4

one

fourths

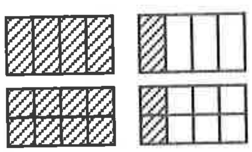


4 fourths

4. $2\frac{1}{2}$

quarters

halves

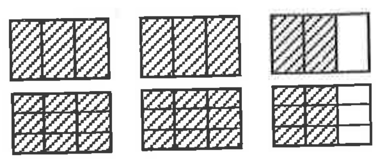


$2\frac{1}{2}$ halves

5. 24

3rds

9ths



24 ninths

> The next two examples use common denominators to divide by a fraction. Express both fractions with a common denominator, then divide the first numerator by the second.

Example 3

$\frac{4}{5} \div \frac{2}{3} \Rightarrow \frac{12}{15} \div \frac{10}{15} \Rightarrow \frac{12}{10} \Rightarrow \frac{6}{5}$ or $1\frac{1}{5}$

Example 4

$1\frac{1}{3} \div \frac{1}{6} \Rightarrow \frac{4}{3} \div \frac{1}{6} \Rightarrow \frac{8}{6} \div \frac{1}{6} \Rightarrow \frac{8}{1}$ or 8

One more way to divide fractions is to use the Giant One from previous work with fractions to create a "Super Giant One." To use a Super Giant One, write the division problem in fraction form, with a fraction in both the numerator and the denominator. Use the reciprocal of the denominator for the numerator and the denominator in the Super Giant One, multiply the fractions as usual, and simplify the resulting fraction when possible.

Example 5

$$\frac{\frac{1}{2}}{\frac{1}{4}} \cdot \frac{\frac{4}{1}}{\frac{4}{1}} = \frac{\frac{4}{2}}{1} = \frac{4}{2} = 2$$

Example 6

$$\frac{\frac{3}{4}}{\frac{1}{6}} \cdot \frac{\frac{6}{1}}{\frac{6}{1}} = \frac{\frac{18}{4}}{1} = \frac{9}{2} = 4\frac{1}{2}$$

Example 7

$$\frac{1\frac{1}{3}}{1\frac{1}{2}} = \frac{\frac{4}{3}}{\frac{3}{2}} \cdot \frac{\frac{2}{3}}{\frac{2}{3}} = \frac{\frac{8}{9}}{1} = \frac{8}{9}$$

Example 8

$$\frac{2}{3} \div \frac{3}{5} \Rightarrow \frac{10}{15} \div \frac{9}{15} \Rightarrow \frac{10}{9}$$

Compared to:

$$\frac{\frac{2}{3}}{\frac{3}{5}} \cdot \frac{\frac{5}{3}}{\frac{5}{3}} = \frac{\frac{10}{9}}{1} = \frac{10}{9} = 1\frac{1}{9}$$

Problems

Complete the division problems below. Use any method.

1. $\frac{3}{7} \div \frac{1}{8}$	2. $1\frac{3}{7} \div \frac{1}{2}$	3. $\frac{4}{7} \div \frac{1}{3}$	4. $1\frac{4}{7} \div \frac{1}{3}$	5. $\frac{6}{7} \div \frac{5}{8}$
6. $\frac{3}{10} \div \frac{5}{7}$	7. $2\frac{1}{3} \div \frac{5}{8}$	8. $7 \div \frac{1}{3}$	9. $1\frac{1}{3} \div \frac{2}{5}$	10. $2\frac{2}{3} \div \frac{3}{4}$
11. $3\frac{1}{3} \div \frac{5}{6}$	12. $1\frac{1}{2} \div \frac{1}{2}$	13. $\frac{5}{8} \div 1\frac{1}{4}$	14. $10\frac{1}{3} \div \frac{1}{6}$	15. $\frac{3}{5} \div 6$

Answers

1. $3\frac{3}{7}$	2. $2\frac{6}{7}$	3. $1\frac{5}{7}$	4. $4\frac{5}{7}$	5. $1\frac{13}{35}$
6. $\frac{21}{50}$	7. $3\frac{11}{15}$	8. 21	9. $3\frac{1}{3}$	10. $3\frac{5}{9}$
11. 4	12. 3	13. $\frac{1}{2}$	14. 62	15. $\frac{1}{10}$

B

OPERATIONS WITH INTEGERS

ADDITION OF INTEGERS

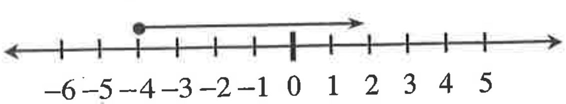
Students review addition of integers using two concrete models: movement along a number line and positive and negative integer tiles.

To add two integers using a number line, start at the first number and then move the appropriate number of spaces to the right or left depending on whether the second number is positive or negative, respectively. Your final location is the sum of the two integers.

To add two integers using integer tiles, a positive number is represented by the appropriate number of (+) tiles and a negative number is represented by the appropriate number of (–) tiles. To add two integers start with a tile representation of the first integer in a diagram and then place into the diagram a tile representative of the second integer. Any equal number of (+) tiles and (–) tiles makes "zero" and can be removed from the diagram. The tiles that remain represent the sum. For additional information, see the Math Notes boxes in Lesson 3.2.3 of the *Core Connections, Course 1* text, or Lesson 2.2.4 of the *Core Connections, Course 2* text.

Example 1

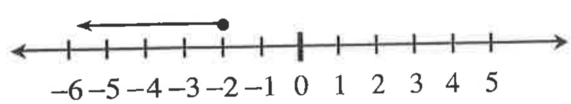
$-4+6$



$-4+6=2$

Example 2

$-2+(-4)$



$-2+(-4)=-6$

Example 3

$5+(-6)$

Start with tiles representing the first number.
+ + + + +

Add to the diagram tiles representing the second number.
+ + + + +
– – – – – –

Circle the zero pairs.
–1 is the answer.
(+ –) (+ –) (+ –) (+ –) (+ –) –

$5+(-6)=-1$

Example 4

$-3+7$

(+ –) (+ –) (+ –) + + + +

$-3+7=4$

ADDITION OF INTEGERS IN GENERAL

When you add integers using the tile model, zero pairs are only formed if the two numbers have different signs. After you circle the zero pairs, you count the uncircled tiles to find the sum. If the signs are the same, no zero pairs are formed, and you find the sum of the tiles. Integers can be added without building models by using the rules below.

- If the signs are the same, add the numbers and keep the same sign.
- If the signs are different, ignore the signs (that is, use the absolute value of each number.) Subtract the number closest to zero from the number farthest from zero. The sign of the answer is the same as the number that is farthest from zero, that is, the number with the greater absolute value.

Example

For $-4 + 2$, -4 is farther from zero on the number line than 2, so subtract: $4 - 2 = 2$.
The answer is -2, since the "4," that is, the number farthest from zero, is negative in the original problem.

Problems

Use either model or the rules above to find these sums.

1. $4+(-2)$	2. $6+(-1)$	3. $7+(-7)$
4. $-10+6$	5. $-8+2$	6. $-12+7$
7. $-5+(-8)$	8. $-10+(-2)$	9. $-11+(-16)$
10. $-8+10$	11. $-7+15$	12. $-26+12$
13. $-3+4+6$	14. $56+17$	15. $7+(-10)+(-3)$
16. $-95+26$	17. $35+(-6)+8$	18. $-113+274$
19. $105+(-65)+20$	20. $-6+2+(-4)+3+5$	21. $5+(-3)+(-2)+(-8)$
22. $-6+(-3)+(-2)+9$	23. $-6+(-3)+9$	24. $20+(-70)$
25. $12+(-7)+(-8)+4+(-3)$	26. $-26+(-13)$	27. $-16+(-8)+9$
28. $12+(-13)+18+(-16)$	29. $50+(-70)+30$	30. $19+(-13)+(-5)+20$

Answers

1. 2	2. 5	3. 0	4. –4	5. –6	6. –5
7. –13	8. –12	9. –27	10. 2	11. 8	12. –14
13. 7	14. 73	15. –6	16. –69	17. 37	18. 161
19. 60	20. 0	21. –8	22. –2	23. 0	24. –50
25. –2	26. –39	27. –15	28. 1	29. 10	30. 20

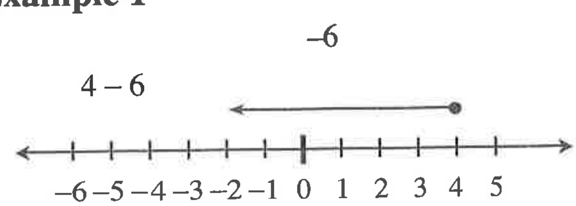
SUBTRACTION OF INTEGERS

Subtraction of integers may also be represented using the concrete models of number lines and (+) and (–) tiles. Subtraction is the opposite of addition so it makes sense to do the opposite actions of addition.

When using the number line, adding a positive integer moves to the right so subtracting a positive integer moves to the left. Adding a negative integer move to the left so subtracting a negative integer moves to the right.

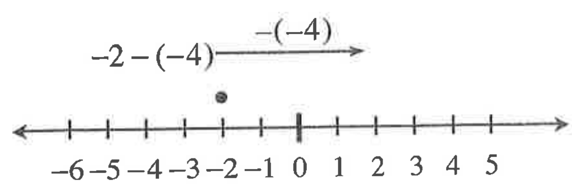
When using the tiles, addition means to place additional tile pieces into the picture and look for zeros to simplify. Subtraction means to remove tile pieces from the picture. Sometimes you will need to place zero pairs in the picture before you have a sufficient number of the desired pieces to remove. For additional information, see the Math Notes box in Lesson 3.2.2 of the *Core Connections, Course 2* text.

Example 1



$4 - 6 = -2$

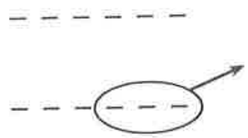
Example 2



$-2 - (-4) = 2$

Example 3

$-6-(-3)$



Build the first integer.

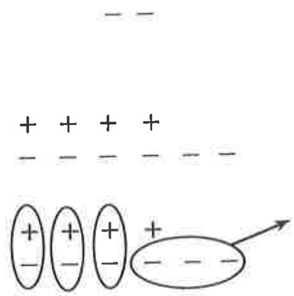
Remove three negatives.

$-6-(-3)=-3$

Three negatives are left.

Example 4

$-2-(-3)$



Build the first integer.

It is not possible to remove three negatives so add some zeros.

Now remove three negatives and circle any zeros.

One positive remains.

$-2-(-3)=1$

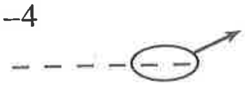
Problems

Find each difference. Use one of the models for at least the first five differences.

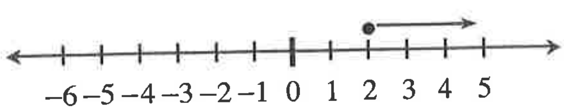
1. $-6-(-2)$
2. $2-(-3)$
3. $6-(-3)$
4. $3-7$
5. $7-(-3)$
6. $7-3$
7. $5-(3)$
8. $-12-(-10)$
9. $-12-10$
10. $12-(-10)$
11. $-6-(-3)-5$
12. $6-(-3)-5$
13. $8-(-8)$
14. $-9-9$
15. $-9-9-(-9)$

Answers (and possible models)

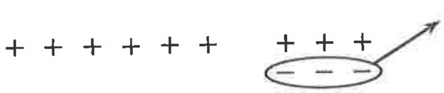
1. -4



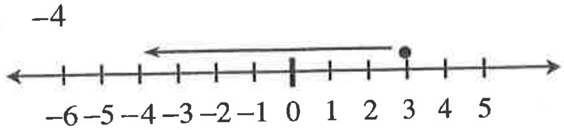
2. 5



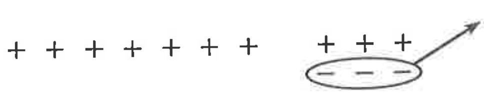
3. 9



4. -4



5. 10

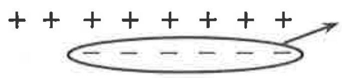


6. 4
7. 2
8. -2
9. -22
10. 22
11. -8
12. 4
13. 16
14. -18
15. -9

CONNECTING ADDITION AND SUBTRACTION

In the next six examples, compare (a) to (b), (c) to (d), and (e) to (f). Notice that examples (a), (c), and (e) are subtraction problems and examples (b), (d), and (f) are addition problems. The answers to each pair of examples are the same. Also notice that the second integers in the pairs are opposites (that is, they are the same distance from zero on opposite sides of the number line) while the first integers in each pair are the same.

a. $2-(-6)$



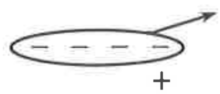
$2-(-6)=8$

b. $2+6$

+ + + + + + + +

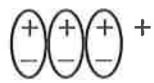
$2+6=8$

c. $-3-(-4)$



$-3-(-4)=1$

d. $-3+4$

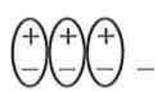


$-3+4=1$

e. $-4-(-3)$

$-4-(-3)=-1$

f. $-4+3$



$-4+3=-1$

You can conclude that subtracting an integer is the same as adding its opposite. This fact is summarized below.

SUBTRACTION OF INTEGERS IN GENERAL

To find the difference of two integers, change the subtraction sign to an addition sign. Next change the sign of the integer you are subtracting, and then apply the rules for addition of integers.

For more information on the rules for subtracting integers, see the Math Notes box in Lesson 3.2.3 of the *Core Connections, Course 2* text.

Examples

Use the rule for subtracting integers stated above to find each difference (that is, subtract).

a. $9-(-12)$ becomes $9+(+12)=21$

b. $-9-(-12)$ becomes $-9+(+12)=3$

c. $-9-12$ becomes $-9+(-12)=-21$

d. $9-12$ becomes $9+(-12)=-3$

Problems

Use the rule stated above to find each difference.

1. $9-(-3)$
2. $9-3$
3. $-9-3$
4. $-9-(-3)$
5. $-14-15$
6. $-16-(-15)$
7. $-40-62$
8. $-40-(-62)$
9. $40-62$
10. $40-(-62)$
11. $-5-(-3)-5-6$
12. $-5-3-(-5)-(-6)$
13. $5-3-(-5)-6$
14. $5-(-4)-6-(-7)$
15. $-125-(-125)-125$
16. $5-(-6)$
17. $12-14$
18. $20-25$
19. $-3-2$
20. $-7-3$
21. $-10-5$
22. $-30-7$
23. $-3-(-3)$
24. $-3-(-4)$
25. $10-(-3)$
26. $5-(-9)$
27. $27-(-7)$
28. $15-32$
29. $-58-37$
30. $-79-(-32)$
31. $-62-81$
32. $-106-242$
33. $47-(-55)$
34. $257-349$
35. $-1010-(-1010)$

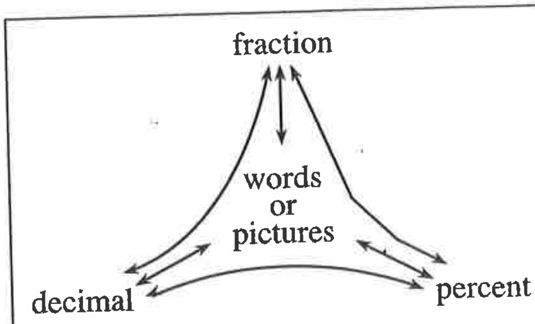
Answers

1. 12	2. 6	3. −12	4. −6	5. −29
6. −1	7. −102	8. 22	9. −22	10. 102
11. −13	12. 3	13. 1	14. 10	15. −125
16. 11	17. −2	18. −5	19. −5	20. −10
21. −15	22. −37	23. 0	24. 1	25. 13
26. 14	27. 34	28. −17	29. −95	30. −47
31. −143	32. −348	33. 102	34. −92	35. 0

C

FRACTION-DECIMAL-PERCENT EQUIVALENTS

Fractions, decimals, and percents are different ways to represent the same portion or number.



Representations of a Portion

For additional information, see the Math Notes boxes in Lessons 3.1.4 and 3.1.5 of the *Core Connections, Course 1* text, Lesson 2.1.2 of the *Core Connections, Course 2* text, or Lesson 1.1.1 of the *Core Connections, Course 3* text. For additional examples and practice, see the *Core Connections, Course 1* Checkpoint 5 materials or the *Core Connections, Course 2* Checkpoint 2 materials.

Examples

Decimal to percent:

Multiply the decimal by 100.

$(0.81)(100) = 81\%$

Fraction to percent:

Write a proportion to find an equivalent fraction using 100 as the denominator. The numerator is the percent.

$$\frac{4}{5} = \frac{x}{100} \text{ so } \frac{4}{5} = \frac{80}{100} = 80\%$$

Decimal to fraction:

Use the digits in the decimal as the numerator.
Use the decimal place value name as the denominator. Simplify as needed.

a. $0.2 = \frac{2}{10} = \frac{1}{5}$ b. $0.17 = \frac{17}{100}$

Percent to decimal:

Divide the percent by 100.

$43\% \div 100 = 0.43$

Percent to fraction:

Use 100 as the denominator. Use the percent as the numerator. Simplify as needed.

$22\% = \frac{22}{100} = \frac{11}{50}$

$56\% = \frac{56}{100} = \frac{14}{25}$

Fraction to decimal:

Divide the numerator by the denominator.

$\frac{3}{8} = 3 \div 8 = 0.375$ $\frac{5}{8} = 5 \div 8 = 0.625$

$\frac{3}{11} = 3 \div 11 = 0.2727\ldots = 0.\overline{27}$

To see the process for converting repeating decimals to fractions, see problem 2-22 in the *Core Connections, Course 2* text, problem 9-105 in the *Core Connections, Course 3* text, or the Math Notes boxes referenced above.

Problems

Convert the fraction, decimal, or percent as indicated.

1. Change $\frac{1}{4}$ to a decimal.
2. Change 50% into a fraction in lowest terms.
3. Change 0.75 to a fraction in lowest terms.
4. Change 75% to a decimal.
5. Change 0.38 to a percent.
6. Change $\frac{1}{5}$ to a percent.
7. Change 0.3 to a fraction.
8. Change $\frac{1}{8}$ to a decimal.
9. Change $\frac{1}{3}$ to a decimal.
10. Change 0.08 to a percent.
11. Change 87% to a decimal.
12. Change $\frac{3}{5}$ to a percent.
13. Change 0.4 to a fraction in lowest terms.
14. Change 65% to a fraction in lowest terms.
15. Change $\frac{1}{9}$ to a decimal.
16. Change 125% to a fraction in lowest terms.
17. Change $\frac{8}{5}$ to a decimal.
18. Change 3.25 to a percent.
19. Change $\frac{1}{16}$ to a decimal. Change the decimal to a percent.
20. Change $\frac{1}{7}$ to a decimal.
21. Change 43% to a fraction. Change the fraction to a decimal.
22. Change 0.375 to a percent. Change the percent to a fraction.
23. Change $\frac{7}{8}$ to a decimal. Change the decimal to a percent.
24. Change $0.\overline{12}$ to a fraction
25. Change $0.\overline{175}$ to a fraction

Answers

1. 0.25
2. $\frac{1}{2}$
3. $\frac{3}{4}$
4. 0.75
5. 38%
6. 20%
7. $\frac{3}{10}$
8. 0.125
9. $0.3\overline{3}$
10. 8%
11. 0.87
12. 60%
13. $\frac{2}{5}$
14. $\frac{13}{20}$
15. $0.1\overline{1}$
16. $\frac{5}{4}$ or $1\frac{1}{4}$
17. 1.6
18. 325%
19. 0.0625; 6.25%
20. $0.\overline{142859}$
21. $\frac{43}{100}$; 0.43
22. $37\frac{1}{2}$%; $\frac{3}{8}$
23. 0.875; 87.5%
24. $\frac{12}{99} = \frac{4}{33}$
25. $\frac{175}{999}$

D

OPERATIONS WITH DECIMALS

ARITHMETIC OPERATIONS WITH DECIMALS

ADDING AND SUBTRACTING DECIMALS: Write the problem in column form with the decimal points in a vertical column. Write in zeros so that all decimal parts of the number have the same number of digits. Add or subtract as with whole numbers. Place the decimal point in the answer aligned with those above.

MULTIPLYING DECIMALS: Multiply as with whole numbers. In the product, the number of decimal places is equal to the total number of decimal places in the factors (numbers you multiplied). Sometimes zeros need to be added to place the decimal point.

DIVIDING DECIMALS: When dividing a decimal by a whole number, place the decimal point in the answer space directly above the decimal point in the number being divided. Divide as with whole numbers. Sometimes it is necessary to add zeros to the number being divided to complete the division.

When dividing decimals or whole numbers by a decimal, the divisor must be multiplied by a power of ten to make it a whole number. The dividend must be multiplied by the same power of ten. Then divide following the same rules for division by a whole number.

For additional information, see the Math Notes boxes in Lesson 5.2.2 of the *Core Connections, Course 1* text, or Lessons 3.3.2 and 3.3.3 of the *Core Connections, Course 2* text. For additional examples and practice, see the *Core Connections, Course 1* Checkpoint 2, Checkpoint 7A, and Checkpoint 8B materials.

Example 1

Add 47.37, 28.9, 14.56, and 7.8.

$$\begin{array}{r} 47.37 \\ 28.9\mathbf{0} \\ 14.56 \\ +\ 7.8\mathbf{0} \\ \hline 98.63 \end{array}$$

Example 2

Subtract 198.76 from 473.2.

$$\begin{array}{r} 473.2\mathbf{0} \\ -\ 198.76 \\ \hline 274.44 \end{array}$$

Example 3

Multiply 27.32 by 14.53.

$$\begin{array}{rl} 27.32 & \text{(2 decimal places)} \\ \text{x } 14.53 & \text{(2 decimal places)} \\ \hline 8196 & \\ 13660 & \\ 10928 & \\ 2732 & \\ \hline 396.9596 & \text{(4 decimal places)} \end{array}$$

Example 4

Multiply 0.37 by 0.0004.

$$\begin{array}{rl} 0.37 & \text{(2 decimal places)} \\ \text{x } 0.0004 & \text{(4 decimal places)} \\ \hline \mathbf{0.000}148 & \text{(6 decimal places)} \end{array}$$

Example 5

Divide 32.4 by 8.

$$\begin{array}{r} 4.05 \\ 8\overline{)32.4\mathbf{0}} \\ 32 \\ \hline 0\ 40 \\ 40 \\ \hline 0 \end{array}$$

Example 6

Divide 27.42 by 1.2. First multiply each number by 10^1 or 10.

$$1.2\overline{)27.42} \Rightarrow 12\overline{)274.2} \Rightarrow \begin{array}{r} 22.85 \\ 12\overline{)274.20} \\ 24 \\ \hline 34 \\ 24 \\ \hline 10\ 2 \\ 9\ 6 \\ \hline 60 \\ 60 \\ \hline 0 \end{array}$$

Problems

1. $4.7 + 7.9$
2. $3.93 + 2.82$
3. $38.72 + 6.7$
4. $58.3 + 72.84$
5. $4.73 + 692$
6. $428 + 7.392$
7. $42.1083 + 14.73$
8. $9.87 + 87.47936$
9. $9.999 + 0.001$
10. $0.0001 + 99.9999$
11. $0.0137 + 1.78$
12. $2.037 + 0.09387$
13. $15.3 + 72.894$
14. $47.9 + 68.073$
15. $289.307 + 15.938$
16. $476.384 + 27.847$
17. $15.38 + 27.4 + 9.076$
18. $48.32 + 284.3 + 4.638$
19. $278.63 + 47.0432 + 21.6$
20. $347.68 + 28.00476 + 84.3$
21. $8.73 - 4.6$
22. $9.38 - 7.5$
23. $8.312 - 6.98$
24. $7.045 - 3.76$
25. $6.304 - 3.68$
26. $8.021 - 4.37$
27. $14 - 7.431$
28. $23 - 15.37$
29. $10 - 4.652$
30. $18 - 9.043$
31. $0.832 - 0.47$
32. $0.647 - 0.39$
33. $1.34 - 0.0538$
34. $2.07 - 0.523$
35. $4.2 - 1.764$
36. $3.8 - 2.406$
37. $38.42 - 32.605$
38. $47.13 - 42.703$
39. $15.368 + 14.4 - 18.5376$
40. $87.43 - 15.687 - 28.0363$
41. $7.34 \cdot 6.4$
42. $3.71 \cdot 4.03$
43. $0.08 \cdot 4.7$
44. $0.04 \cdot 3.75$
45. $41.6 \cdot 0.302$
46. $9.4 \cdot 0.0053$
47. $3.07 \cdot 5.4$
48. $4.023 \cdot 3.02$
49. $0.004 \cdot 0.005$
50. $0.007 \cdot 0.0004$
51. $0.235 \cdot 0.43$
52. $4.32 \cdot 0.0072$
53. $0.0006 \cdot 0.00013$
54. $0.0005 \cdot 0.00026$
55. $8.38 \cdot 0.0001$
56. $47.63 \cdot 0.000001$
57. $0.078 \cdot 3.1$
58. $0.043 \cdot 4.2$
59. $350 \cdot 0.004$
60. $421 \cdot 0.00005$

Divide. Round answers to the hundredth, if necessary.

61. $14.3 \div 8$	62. $18.32 \div 5$	63. $147.3 \div 6$
64. $46.36 \div 12$	65. $100.32 \div 24$	66. $132.7 \div 28$
67. $47.3 \div 0.002$	68. $53.6 \div 0.004$	69. $500 \div 0.004$
70. $420 \div 0.05$	71. $1.32 \div 0.032$	72. $3.486 \div 0.012$
73. $46.3 \div 0.011$	74. $53.7 \div 0.023$	75. $25.46 \div 5.05$
76. $26.35 \div 2.2$	77. $6.042 \div 0.006$	78. $7.035 \div 0.005$
79. $207.3 \div 4.4$	80. $306.4 \div 3.2$	

Answers

1. 12.6	2. 6.75	3. 45.42	4. 131.14	5. 696.73
6. 435.392	7. 56.8383	8. 97.34936	9. 10.000	10. 100.0000
11. 1.7937	12. 2.13087	13. 88.194	14. 115.973	15. 305.245
16. 504.231	17. 51.856	18. 337.258	19. 347.2732	20. 459.98476
21. 4.13	22. 1.88	23. 1.332	24. 3.285	25. 2.624
26. 3.651	27. 6.569	28. 7.63	29. 5.348	30. 8.957
31. 0.362	32. 0.257	33. 1.2862	34. 1.547	35. 2.436
36. 1.394	37. 5.815	38. 4.427	39. 11.2304	40. 43.7067
41. 46.976	42. 14.9513	43. 0.376	44. 0.15	45. 12.5632
46. 0.04982	47. 16.578	48. 12.14946	49. 0.000020	50. 0.0000028
51. 0.10105	52. 0.031104	53. 0.000000078	54. 0.000000130	55. 0.000838
56. 0.0004763	57. 0.2418	58. 0.1806	59. 1.4	60. 0.02105
61. 1.7875 or 1.79	62. 3.664 or 3.66	63. 24.55	64. $3.86\overline{3}$ or 3.86	65. 4.18
66. 4.74	67. 23,650	68. 13,400	69. 125,000	70. 8400
71. 41.25	72. 29.05	73. 4209.09	74. 2334.78	75. 5.04
76. 11.98	77. 1007	78. 1407	79. 47.11	80. 95.75

MULTIPLYING DECIMALS AND PERCENTS

Understanding how many places to move the decimal point in a decimal multiplication problem is connected to the multiplication of fractions and place value.

Computations involving calculating "a percent of a number" are simplified by changing the percent to a decimal.

Example 1

Multiply $(0.2)\cdot(0.3)$.

In fractions this means $\frac{2}{10}\cdot\frac{3}{10}\Rightarrow\frac{6}{100}$.

Knowing that the answer must be in the hundredths place tells you how many places to move the decimal point (to the left) without using the fractions.

(tenths)(tenths) = hundredths

Therefore move two places.

$$\begin{array}{r} 0.2 \\ \underline{\times\ 0.3} \\ 0.06 \end{array}$$

Example 2

Multiply $(1.7)\cdot(0.03)$.

In fractions this means $\frac{17}{10}\cdot\frac{3}{100}\Rightarrow\frac{51}{1000}$.

Knowing that the answer must be in the thousandths place tells you how many places to move the decimal point (to the left) without using the fractions.

(tenths)(hundredths) = thousandths

Therefore move three places.

$$\begin{array}{r} 1.7 \\ \underline{\times\ 0.03} \\ 0.051 \end{array}$$

Example 3

Calculate 17% of 32.5 without using a calculator.

Since $17\% = \frac{17}{100} = 0.17$,

17% of $32.5 \Rightarrow (0.17)\cdot(32.5)$

$\Rightarrow 5.525$

$$\begin{array}{r} 32.5 \\ \underline{\times\ 0.17} \\ 2275 \\ \underline{3250} \\ 5.525 \end{array}$$

Problems

Identify the number of places to the left to move the decimal point in the product. Do not compute the product.

1.	$(0.3) \cdot (0.5)$	2.	$(1.5) \cdot (0.12)$	3.	$(1.23) \cdot (2.6)$
4.	$(0.126) \cdot (3.4)$	5.	$17 \cdot (32.016)$	6.	$(4.32) \cdot (3.1416)$

Compute without using a calculator.

7.	$(0.8) \cdot (0.03)$	8.	$(3.2) \cdot (0.3)$	9.	$(1.75) \cdot (0.09)$
10.	$(4.5) \cdot (3.2)$	11.	$(1.8) \cdot (0.032)$	12.	$(7.89) \cdot (6.3)$
13.	8% of 540	14.	70% of 478	15.	37% of 4.7
16.	17% of 96	17.	15% of 4.75	18.	130% of 42

Answers

1.	2	2.	3	3.	3
4.	4	5.	3	6.	6
7.	0.024	8.	0.96	9.	0.1575
10.	14.4	11.	0.0576	12.	49.707
13.	43.2	14.	334.6	15.	1.739
16.	16.32	17.	0.7125	18.	54.6

E

CALCULATING AND USING PERCENTS

Students also calculate percentages by composition and decomposition, that is, breaking numbers into parts, and then adding or subtracting the results. This method is particularly useful for doing mental calculations. A percent ruler is also used for problems when you need to find the percent or the whole.

For additional information, see the Math Notes box in Lesson 9.2.4 of the *Core Connections, Course 1* text.

Knowing quick methods to calculate 10% of a number and 1% of a number will help you to calculate other percents by composition. Use the fact that $10\% = \frac{1}{10}$ and $1\% = \frac{1}{100}$.

Example 1

To calculate 32% of 40, you can think of 3(10% of 40) + 2(1% of 40).

10% of 40 $\Rightarrow \frac{1}{10}$ of 40 = 4 and 1% of 40 $\Rightarrow \frac{1}{100}$ of 40 = 0.4 so

32% of 40 $\Rightarrow 3(4) + 2(0.4) \Rightarrow 12 + 0.8 = 12.8$

Example 2

To calculate 9% of 750, you can think of 10% of 750 – 1% of 750.

10% of 750 $\Rightarrow \frac{1}{10}$ of 750 = 75 and 1% of 750 $\Rightarrow \frac{1}{100}$ of 750 = 7.5 so

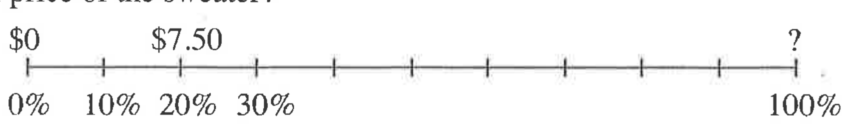
9% of 750 $\Rightarrow 75 - 7.5 = 67.5$

Other common percents such as $50\% = \frac{1}{2}$, $25\% = \frac{1}{4}$, $75\% = \frac{3}{4}$, $20\% = \frac{1}{5}$ may also be used.

Students also use a percent ruler to find missing parts in percent problems.

Example 3

Jana saved \$7.50 of the original price of a sweater when it was on sale for 20% off. What was the original price of the sweater?



If every 20% is \$7.50, the other four 20% parts $(4 \cdot 7.50 + 7.50)$ find that 100% is \$37.50.

Example 4

To calculate 17% of 123.4 convert the percent to a decimal and then use direct computation by hand or with a calculator.

17% of $123.4 \Rightarrow \frac{17}{100}(123.4) \Rightarrow 0.17(123.4) = 20.978$

Problems

Solve each problem without a calculator. Show your work or reasoning.

1. What is 22% of 60?
2. What is 29% of 500?
3. What is 41% of 82?
4. What is 8% of 65?
5. 45 is 30% of what number?
6. $1.50 is 25% of what amount?
7. 60 is what % of 80?
8. What is 15% of 32?
9. What percent is 16 out of 20?
10. $10 is what percent of $25?
11. 30% of what number is 27
12. 15% of what number is 24?

Compute by any method. Round to the nearest cent as appropriate.

13. 17% of 125.8
14. 32% of 4.825
15. 125% of 49
16. 6.5% of 48
17. 5.2% of 8.7
18. 7% of $14.95
19. 9% of $32.95
20. 1.5% of $3200
21. 1.65% of $10,500
22. 120% of 67
23. 167% of 45.8
24. 15% of $24.96
25. 20% of 32
26. 2.5% of 1400
27. 3.6% of 2450

Answers

1.	13.2	2.	145	3.	33.62	4.	52
5.	150	6.	$6.00	7.	75%	8.	4.8
9.	80%	10.	40%	11.	90	12.	160
13.	21.386	14.	1.544	15.	61.25	16.	3.12
17.	0.4524	18.	$1.05	19.	$2.97	20.	$48
21.	$173.25	22.	80.4	23.	76.486	24.	$3.74
25.	6.4	26.	35	27.	88.2		

F

RATES AND UNIT RATES

Rate of change is a ratio that describes how one quantity is changing with respect to another. Unit rate is a rate that compares the change in one quantity to a one-unit change in another quantity. Some examples of rates are miles per hour and price per pound. If 16 ounces of flour cost \$0.80 then the unit cost, that is the cost per one once, is $\frac{\$0.80}{16} = \$0.05$.

For additional information see the Math Notes boxes in Lesson 7.1.3 of the *Core Connections, Course 1* text, Lesson 4.2.4 of the *Core Connections, Course 2* text, or Lesson 7.2.5 of the *Core Connections, Course 3* text. For additional examples and practice, see the *Core Connections, Course 2* Checkpoint 9 materials or the *Core Connections, Course 3* Checkpoint 3 materials.

Example 1

A rice recipe uses 6 cups of rice for 15 people. At the same rate, how much rice is needed for 40 people?

The rate is: $\frac{6 \text{ cups}}{15 \text{ people}}$ so we need to solve $\frac{6}{15} = \frac{x}{40}$.

The multiplier needed for the Giant One is $\frac{40}{15}$ or $2\frac{2}{3}$.

Using that multiplier yields $\frac{6}{15} \cdot \frac{2\frac{2}{3}}{2\frac{2}{3}} = \frac{16}{40}$ so 16 cups of rice is needed.

Note that the equation $\frac{6}{15} = \frac{x}{40}$ can also be solved using proportions.

Example 2

Arrange these rates from least to greatest:

30 miles in 25 minutes   60 miles in one hour   70 miles in $1\frac{2}{3}$ hr

Changing each rate to a common denominator of 60 minutes yields:

$\frac{30 \text{ mi}}{25 \text{ min}} = \frac{x}{60} \Rightarrow \frac{30}{25} \cdot \frac{2.4}{2.4} = \frac{72}{60} \frac{\text{mi}}{\text{min}}$ $\frac{60 \text{ mi}}{1 \text{ hr}} = \frac{60 \text{ mi}}{60 \text{ min}}$ $\frac{70 \text{ mi}}{1\frac{2}{3} \text{ hr}} = \frac{70 \text{ mi}}{100 \text{ min}} = \frac{x}{60} \Rightarrow \frac{70}{100} \cdot \frac{0.6}{0.6} = \frac{42 \text{ mi}}{60 \text{ min}}$

So the order from least to greatest is: 70 miles in $1\frac{2}{3}$ hr < 60 miles in one hour < 30 miles in 25 minutes. Note that by using 60 minutes (one hour) for the common unit to compare speeds, we can express each rate as a unit rate: 42 mph, 60 mph, and 72 mph.

Example 3

A train in France traveled 932 miles in 5 hours. What is the unit rate in miles per hour?

Unit rate means the denominator needs to be 1 hour so: $\frac{932 \text{ mi}}{5 \text{ hr}} = \frac{x}{1 \text{ hr}}$. Solving by using a Giant One of $\frac{0.2}{0.2}$ or simple division yields $x = 186.4$ miles per hour.

Problems

Solve each rate problem below. Explain your method.

1. Balvina knows that 6 cups of rice will make enough Spanish rice to feed 15 people. She needs to know how many cups of rice are needed to feed 135 people.
2. Elaine can plant 6 flowers in 15 minutes. How long will it take her to plant 30 flowers at the same rate?
3. A plane travels 3400 miles in 8 hours. How far would it travel in 6 hours at this rate?
4. Shane rode his bike for 2 hours and traveled 12 miles. At this rate, how long would it take him to travel 22 miles?
5. Selina's car used 15.6 gallons of gas to go 234 miles. At this rate, how many gallons would it take her to go 480 miles?
6. Arrange these readers from fastest to slowest: Abel read 50 pages in 45 minutes, Brian read 90 pages in 75 minutes, and Charlie read 175 pages in 2 hours.
7. Arrange these lunch buyers from greatest to least assuming they buy lunch 5 days per week: Alice spends $3 per day, Betty spends $25 every two weeks, and Cindy spends $75 per month.
8. A train in Japan can travel 813.5 miles in 5 hours. Find the unit rate in miles per hour.
9. An ice skater covered 1500 meters in 106 seconds. Find his unit rate in meters per second.
10. A cellular company offers a price of $19.95 for 200 minutes. Find the unit rate in cost per minute.
11. A car traveled 200 miles on 8 gallons of gas. Find the unit rate of miles per gallon and the unit rate of gallons per mile.
12. Lee's paper clip chain is 32 feet long. He is going to add paper clips continually for the next eight hours. At the end of eight hours the chain is 80 feet long. Find the unit rate of growth in feet per hour.

Answers

1.	54 cups	2.	75 min	3.	2550 miles	4.	$3\frac{2}{3}$ hr
5.	32 gallons	6.	C, B, A	7.	C, A, B	8.	162.7 mi/hr
9.	≈ 14.15 m/s	10.	$\approx \$0.10$/min	11.	25 m/g; $\frac{1}{25}$ g/m	12.	6 ft/hr

DISTANCE, RATE, AND TIME

Distance (d) equals the product of the rate of speed (r) and the time (t). This relationship is shown below in three forms:

$$d = r \cdot t \qquad r = \frac{d}{t} \qquad t = \frac{d}{r}$$

It is important that the units of measure are consistent. For additional information see the Math Notes box in Lesson 8.3.2 of the *Core Connections, Course 1* text.

Example 1

Find the rate of speed of a passenger car if the distance traveled is 572 miles and the time elapsed is 11 hours.

$$572 \text{ miles} = r \cdot 11 \text{ hours} \Rightarrow \frac{572 \text{ miles}}{11 \text{ hours}} = r \Rightarrow 52 \text{ miles/hour} = \text{rate}$$

Example 2

Find the distance traveled by a train at 135 miles per hour for 40 minutes.

The units of time are not the same so we need to change 40 minutes into hours. $\frac{40}{60} = \frac{2}{3}$ hour.

$$d = (135 \text{ miles/hour})(\tfrac{2}{3} \text{ hour}) \Rightarrow d = 90 \text{ miles}$$

Example 3

The Central Middle School hamster race is fast approaching. Fred said that his hamster traveled 60 feet in 90 seconds and Wilma said she timed for one minute and her hamster traveled 12 yards. Which hamster has the fastest rate?

rate = $\frac{\text{distance}}{\text{time}}$ but all the measurements need to be in the same units. In this example, we use feet and minutes.

Fred's hamster: rate = $\frac{60 \text{ feet}}{1.5 \text{ minutes}}$ $\Rightarrow$ rate = 40 feet/minute

Wilma's hamster: rate = $\frac{36 \text{ feet}}{1 \text{ minute}}$ $\Rightarrow$ rate = 36 feet/minute

Fred's hamster is faster.

Problems

Solve the following problems.

1. Find the time if the distance is 157.5 miles and the speed is 63 mph.
2. Find the distance if the speed is 67 mph and the time is 3.5 hours.
3. Find the rate if the distance is 247 miles and the time is 3.8 hours.
4. Find the distance if the speed is 60 mph and the time is 1 hour and 45 minutes.
5. Find the rate in mph if the distance is 3.5 miles and the time is 20 minutes.
6. Find the time in minutes if the distance is 2 miles and the rate is 30 mph.
7. Which rate is faster? A: 60 feet in 90 seconds or B: 60 inches in 5 seconds
8. Which distance is longer? A: 4 feet/second for a minute or B: 3 inches/min for an hour
9. Which time is shorter? A: 4 miles at 60 mph or B: 6 miles at 80 mph

Answers

1.	2.5 hr	2.	234.5 mi	3.	65 mph	4.	105 miles	5.	10.5 mph
6.	4 min	7.	B	8.	A	9.	A		